Chapter 2

Motion in One Dimension

▲ *One of the physical quantities we will study in this chapter is the velocity of an object moving in a straight line. Downhill skiers can reach velocities with a magnitude greater than 100 km/h. (Jean Y. Ruszniewski/Getty Images)*

CHAPTER OUTLINE

As a first step in studying classical mechanics, we describe motion in terms of space and time while ignoring the agents that caused that motion. This portion of classical mechanics is called *kinematics.* (The word *kinematics* has the same root as *cinema.* Can you see why?) In this chapter we consider only motion in one dimension, that is, motion along a straight line. We first define position, displacement, velocity, and acceleration. Then, using these concepts, we study the motion of objects traveling in one dimension with a constant acceleration.

From everyday experience we recognize that motion represents a continuous change in the position of an object. In physics we can categorize motion into three types: translational, rotational, and vibrational. A car moving down a highway is an example of translational motion, the Earth's spin on its axis is an example of rotational motion, and the back-and-forth movement of a pendulum is an example of vibrational motion. In this and the next few chapters, we are concerned only with translational motion. (Later in the book we shall discuss rotational and vibrational motions.)

In our study of translational motion, we use what is called the **particle model—** we describe the moving object as a *particle* regardless of its size. In general, **a particle is a point-like object—that is, an object with mass but having infinitesimal size.** For example, if we wish to describe the motion of the Earth around the Sun, we can treat the Earth as a particle and obtain reasonably accurate data about its orbit. This approximation is justified because the radius of the Earth's orbit is large compared with the dimensions of the Earth and the Sun. As an example on a much smaller scale, it is possible to explain the pressure exerted by a gas on the walls of a container by treating the gas molecules as particles, without regard for the internal structure of the molecules.

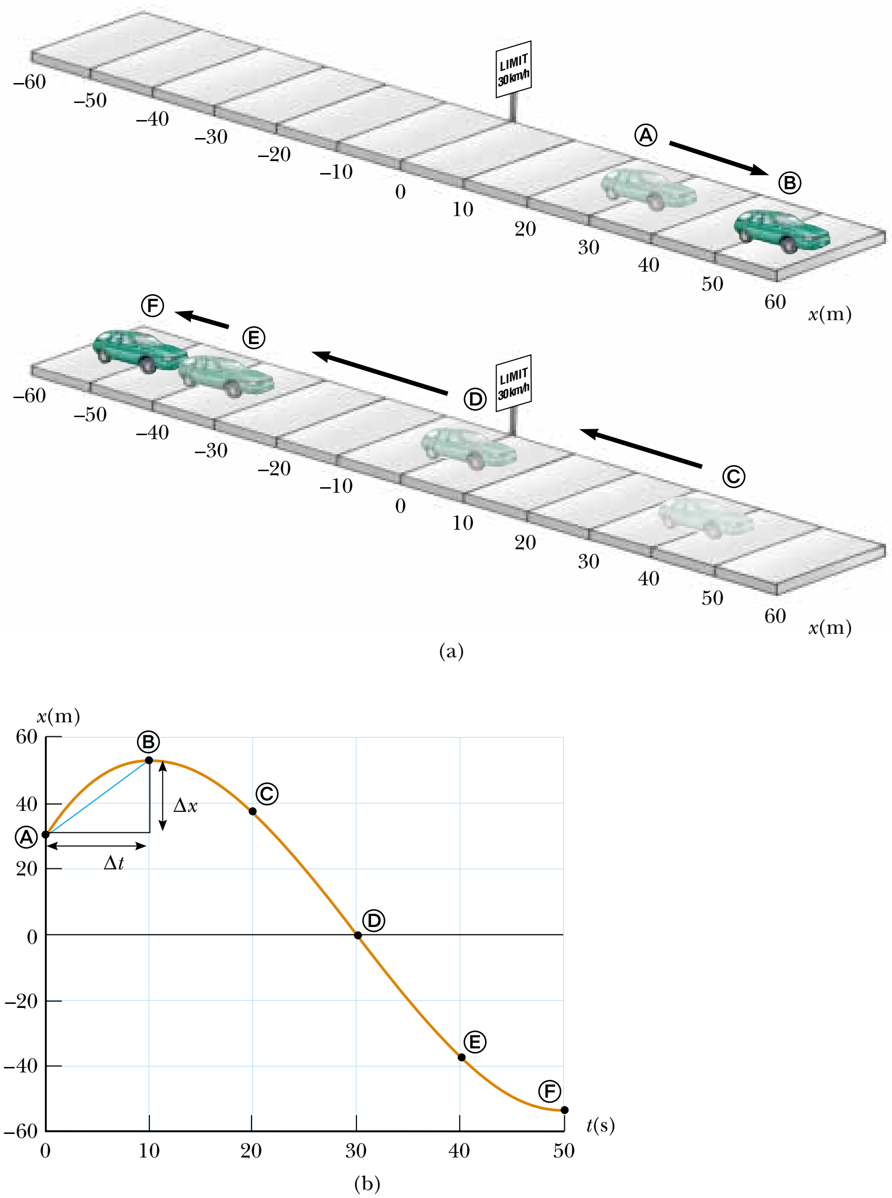
2.1 Position, Velocity, and Speed

Position

The motion of a particle is completely known if the particle's position in space is known at all times. A particle's **position** is the location of the particle with respect to a chosen reference point that we can consider to be the origin of a coordinate system.

Consider a car moving back and forth along the x axis as in Figure 2.1a. When we begin collecting position data, the car is 30 m to the right of a road sign, which we will use to identify the reference position $x = 0$. (Let us assume that all data in this example are known to two significant figures. To convey this information, we should report the initial position as 3.0×10^1 m. We have written this value in the simpler form 30 m to make the discussion easier to follow.) We will use the particle model by identifying some point on the car, perhaps the front door handle, as a particle representing the entire car.

We start our clock and once every 10 s note the car's position relative to the sign at $x = 0$. As you can see from Table 2.1, the car moves to the right (which we have



Active Figure 2.1 (a) A car moves back and forth along a straight line taken to be the x axis. Because we are interested only in the car's translational motion, we can model it as a particle. (b) Position–time graph for the motion of the "particle."

At the Active Figures link at http://www.pse6.com, you can move each of the six points Ⓐ through Ⓕ and observe the motion of the car pictorially and graphically as it follows a smooth path through the six points.

Table 2.1

Position of the Car at Various Times

Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

defined as the positive direction) during the first 10 s of motion, from position Ⓐ to position Ⓑ. After Ⓑ, the position values begin to decrease, suggesting that the car is backing up from position Ⓑ through position Ⓕ. In fact, at Ⓓ, 30 s after we start measuring, the car is alongside the road sign (see Figure 2.1a) that we are using to mark our origin of coordinates. It continues moving to the left and is more than 50 m to the left of the sign when we stop recording information after our sixth data point. A graphical representation of this information is presented in Figure 2.1b. Such a plot is called a *position–time graph.*

Displacement

Given the data in Table 2.1, we can easily determine the change in position of the car for various time intervals. The **displacement** of a particle is defined as its change in position in some time interval. As it moves from an initial position x_i to a final position x_f, the displacement of the particle is given by $x_f - x_i$. We use the Greek letter delta (Δ) to denote the *change* in a quantity. Therefore, we write the displacement, or change in position, of the particle as

$$\Delta x \equiv x_f - x_i \qquad (2.1)$$

From this definition we see that Δx is positive if x_f is greater than x_i and negative if x_f is less than x_i.

It is very important to recognize the difference between displacement and distance traveled. **Distance** is the length of a path followed by a particle. Consider, for example, the basketball players in Figure 2.2. If a player runs from his own basket down the court to the other team's basket and then returns to his own basket, the *displacement* of the player during this time interval is zero, because he ended up at the same point as he started. During this time interval, however, he covered a *distance* of twice the length of the basketball court.

Displacement is an example of a vector quantity. Many other physical quantities, including position, velocity, and acceleration, also are vectors. In general, **a vector quantity requires the specification of both direction and magnitude.** By contrast, **a scalar quantity has a numerical value and no direction.** In this chapter, we use positive (+) and negative (−) signs to indicate vector direction. We can do this because the chapter deals with one-dimensional motion only; this means that any object we study can be moving only along a straight line. For example, for horizontal motion let us arbitrarily specify to the right as being the positive direction. It follows that any object always moving to the right undergoes a positive displacement $\Delta x > 0$, and any object moving to the left undergoes a negative displacement, so that $\Delta x < 0$. We shall treat vector quantities in greater detail in Chapter 3.

For our basketball player in Figure 2.2, if the trip from his own basket to the opposing basket is described by a displacement of $+28$ m, the trip in the reverse direction represents a displacement of -28 m. Each trip, however, represents a distance of 28 m, because distance is a scalar quantity. The total distance for the trip down the court and back is 56 m. Distance, therefore, is always represented as a positive number, while displacement can be either positive or negative.

There is one very important point that has not yet been mentioned. Note that the data in Table 2.1 results only in the six data points in the graph in Figure 2.1b. The smooth curve drawn through the six points in the graph is only a *possibility* of the actual motion of the car. We only have information about six instants of time—we have no idea what happened in between the data points. The smooth curve is a *guess* as to what happened, but keep in mind that it is *only* a guess.

If the smooth curve does represent the actual motion of the car, the graph contains information about the entire 50-s interval during which we watch the car move. It is much easier to see changes in position from the graph than from a verbal description or even a table of numbers. For example, it is clear that the car was covering more ground during the middle of the 50-s interval than at the end. Between positions Ⓒ and Ⓓ, the car traveled almost 40 m, but during the last 10 s, between positions Ⓔ and Ⓕ, it moved less than half that far. A common way of comparing these different motions is to divide the displacement Δx that occurs between two clock readings by the length of that particular time interval Δt. This turns out to be a very useful ratio, one that we shall use many times. This ratio has been given a special name—*average velocity*. **The average velocity $\bar{v}_x$ of a particle is defined as the**

Ken White/Allsport/Getty Images

Figure 2.2 On this basketball court, players run back and forth for the entire game. The distance that the players run over the duration of the game is nonzero. The displacement of the players over the duration of the game is approximately zero because they keep returning to the same point over and over again.

particle's displacement Δx divided by the time interval Δt during which that displacement occurs:

$$\overline{v}_x \equiv \frac{\Delta x}{\Delta t} \tag{2.2}$$

Average velocity

where the subscript x indicates motion along the x axis. From this definition we see that average velocity has dimensions of length divided by time (L/T)—meters per second in SI units.

The average velocity of a particle moving in one dimension can be positive or negative, depending on the sign of the displacement. (The time interval Δt is always positive.) If the coordinate of the particle increases in time (that is, if $x_f > x_i$), then Δx is positive and $\overline{v}_x = \Delta x/\Delta t$ is positive. This case corresponds to a particle moving in the positive x direction, that is, toward larger values of x. If the coordinate decreases in time (that is, if $x_f < x_i$) then Δx is negative and hence $\overline{v}_x$ is negative. This case corresponds to a particle moving in the negative x direction.

We can interpret average velocity geometrically by drawing a straight line between any two points on the position–time graph in Figure 2.1b. This line forms the hypotenuse of a right triangle of height Δx and base Δt. The slope of this line is the ratio $\Delta x/\Delta t$, which is what we have defined as average velocity in Equation 2.2. For example, the line between positions Ⓐ and Ⓑ in Figure 2.1b has a slope equal to the average velocity of the car between those two times, $(52\text{ m} - 30\text{ m})/(10\text{ s} - 0) = 2.2\text{ m/s}$.

In everyday usage, the terms *speed* and *velocity* are interchangeable. In physics, however, there is a clear distinction between these two quantities. Consider a marathon runner who runs more than 40 km, yet ends up at his starting point. His total displacement is zero, so his average velocity is zero! Nonetheless, we need to be able to quantify how fast he was running. A slightly different ratio accomplishes this for us. The **average speed** of a particle, a scalar quantity, is defined as **the total distance traveled divided by the total time interval required to travel that distance:**

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}} \tag{2.3}$$

Average speed

The SI unit of average speed is the same as the unit of average velocity: meters per second. However, unlike average velocity, average speed has no direction and hence carries no algebraic sign. Notice the distinction between average velocity and average speed—average velocity (Eq. 2.2) is the *displacement* divided by the time interval, while average speed (Eq. 2.3) is the *distance* divided by the time interval.

Knowledge of the average velocity or average speed of a particle does not provide information about the details of the trip. For example, suppose it takes you 45.0 s to travel 100 m down a long straight hallway toward your departure gate at an airport. At the 100-m mark, you realize you missed the rest room, and you return back 25.0 m along the same hallway, taking 10.0 s to make the return trip. The magnitude of the average *velocity* for your trip is $+75.0\text{ m}/55.0\text{ s} = +1.36\text{ m/s}$. The average *speed* for your trip is $125\text{ m}/55.0\text{ s} = 2.27\text{ m/s}$. You may have traveled at various speeds during the walk. Neither average velocity nor average speed provides information about these details.

PITFALL PREVENTION

2.1 Average Speed and Average Velocity

The magnitude of the average velocity is *not* the average speed. For example, consider the marathon runner discussed here. The magnitude of the average velocity is zero, but the average speed is clearly not zero.

Quick Quiz 2.1 Under which of the following conditions is the magnitude of the average velocity of a particle moving in one dimension smaller than the average speed over some time interval? (a) A particle moves in the $+x$ direction without reversing. (b) A particle moves in the $-x$ direction without reversing. (c) A particle moves in the $+x$ direction and then reverses the direction of its motion. (d) There are no conditions for which this is true.

Example 2.1 Calculating the Average Velocity and Speed

Find the displacement, average velocity, and average speed of the car in Figure 2.1a between positions Ⓐ and Ⓕ.

Solution From the position–time graph given in Figure 2.1b, note that $x_A = 30$ m at $t_A = 0$ s and that $x_F = -53$ m at $t_F = 50$ s. Using these values along with the definition of displacement, Equation 2.1, we find that

$$\Delta x = x_F - x_A = -53 \text{ m} - 30 \text{ m} = \boxed{-83 \text{ m}}$$

This result means that the car ends up 83 m in the negative direction (to the left, in this case) from where it started. This number has the correct units and is of the same order of magnitude as the supplied data. A quick look at Figure 2.1a indicates that this is the correct answer.

It is difficult to estimate the average velocity without completing the calculation, but we expect the units to be meters per second. Because the car ends up to the left of where we started taking data, we know the average velocity must be negative. From Equation 2.2,

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{x_F - x_A}{t_F - t_A}$$

$$= \frac{-53 \text{ m} - 30 \text{ m}}{50 \text{ s} - 0 \text{ s}} = \frac{-83 \text{ m}}{50 \text{ s}}$$

$$= \boxed{-1.7 \text{ m/s}}$$

We cannot unambiguously find the average speed of the car from the data in Table 2.1, because we do not have information about the positions of the car between the data points. If we adopt the assumption that the details of the car's position are described by the curve in Figure 2.1b, then the distance traveled is 22 m (from Ⓐ to Ⓑ) plus 105 m (from Ⓑ to Ⓕ) for a total of 127 m. We find the car's average speed for this trip by dividing the distance by the total time (Eq. 2.3):

$$\text{Average speed} = \frac{127 \text{ m}}{50 \text{ s}} = \boxed{2.5 \text{ m/s}}$$

2.2 Instantaneous Velocity and Speed

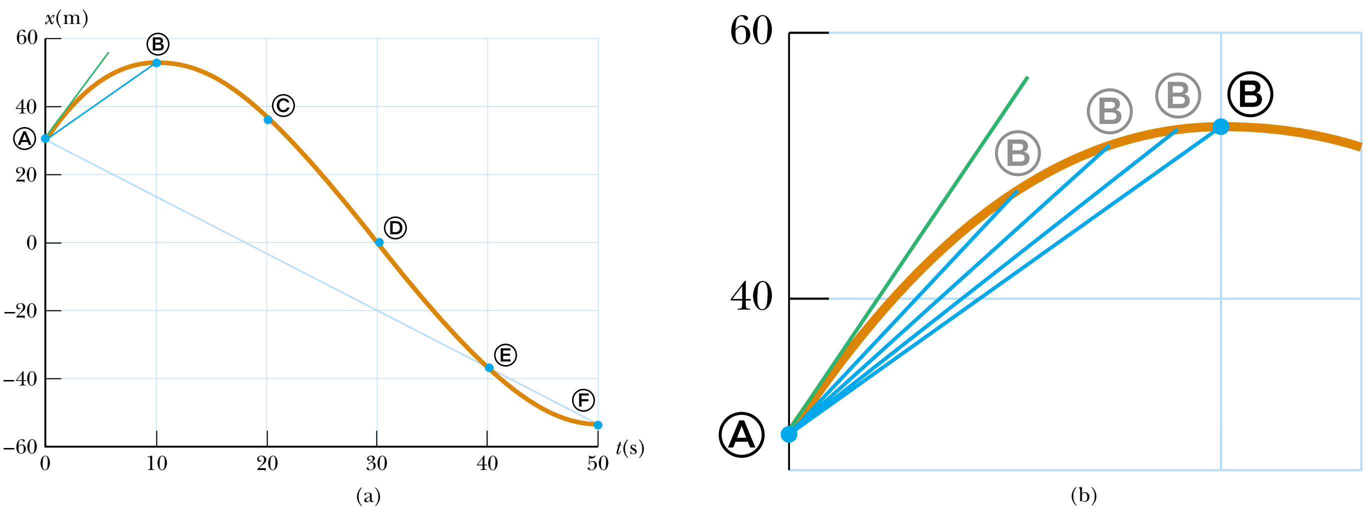
Often we need to know the velocity of a particle at a particular instant in time, rather than the average velocity over a finite time interval. For example, even though you might want to calculate your average velocity during a long automobile trip, you would be especially interested in knowing your velocity at the *instant* you noticed the police car parked alongside the road ahead of you. In other words, you would like to be able to specify your velocity just as precisely as you can specify your position by noting what is happening at a specific clock reading—that is, at some specific instant. It may not be immediately obvious how to do this. What does it mean to talk about how fast something is moving if we "freeze time" and talk only about an individual instant? This is a subtle point not thoroughly understood until the late 1600s. At that time, with the invention of calculus, scientists began to understand how to describe an object's motion at any moment in time.

To see how this is done, consider Figure 2.3a, which is a reproduction of the graph in Figure 2.1b. We have already discussed the average velocity for the interval during which the car moved from position Ⓐ to position Ⓑ (given by the slope of the dark blue line) and for the interval during which it moved from Ⓐ to Ⓕ (represented by the slope of the light blue line and calculated in Example 2.1). Which of these two lines do you think is a closer approximation of the initial velocity of the car? The car starts out by moving to the right, which we defined to be the positive direction. Therefore, being positive, the value of the average velocity during the Ⓐ to Ⓑ interval is more representative of the initial value than is the value of the average velocity during the Ⓐ to Ⓕ interval, which we determined to be negative in Example 2.1. Now let us focus on the dark blue line and slide point Ⓑ to the left along the curve, toward point Ⓐ, as in Figure 2.3b. The line between the points becomes steeper and steeper, and as the two points become extremely close together, the line becomes a tangent line to the curve, indicated by the green line in Figure 2.3b. The slope of this tangent line

PITFALL PREVENTION

2.2 Slopes of Graphs

In any graph of physical data, the *slope* represents the ratio of the change in the quantity represented on the vertical axis to the change in the quantity represented on the horizontal axis. Remember that *a slope has units* (unless both axes have the same units). The units of slope in Figure 2.1b and Figure 2.3 are m/s, the units of velocity.



Active Figure 2.3 (a) Graph representing the motion of the car in Figure 2.1. (b) An enlargement of the upper-left-hand corner of the graph shows how the blue line between positions Ⓐ and Ⓑ approaches the green tangent line as point Ⓑ is moved closer to point Ⓐ.

At the Active Figures link at* http://www.pse6.com, *you can move point Ⓑ as suggested in (b) and observe the blue line approaching the green tangent line.

represents the velocity of the car at the moment we started taking data, at point Ⓐ. What we have done is determine the *instantaneous velocity* at that moment. In other words, **the instantaneous velocity v_x equals the limiting value of the ratio $\Delta x/\Delta t$ as Δt approaches zero:**[1]

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \tag{2.4}$$

In calculus notation, this limit is called the *derivative* of x with respect to t, written dx/dt:

Instantaneous velocity

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \tag{2.5}$$

The instantaneous velocity can be positive, negative, or zero. When the slope of the position–time graph is positive, such as at any time during the first 10 s in Figure 2.3, v_x is positive—the car is moving toward larger values of x. After point Ⓑ, v_x is negative because the slope is negative—the car is moving toward smaller values of x. At point Ⓑ, the slope and the instantaneous velocity are zero—the car is momentarily at rest.

From here on, we use the word *velocity* to designate instantaneous velocity. When it is *average velocity* we are interested in, we shall always use the adjective *average*.

The **instantaneous speed** of a particle is defined as the magnitude of its instantaneous velocity. As with average speed, instantaneous speed has no direction associated with it and hence carries no algebraic sign. For example, if one particle has an instantaneous velocity of $+25$ m/s along a given line and another particle has an instantaneous velocity of -25 m/s along the same line, both have a speed[2] of 25 m/s.

PITFALL PREVENTION

2.3 Instantaneous Speed and Instantaneous Velocity

In Pitfall Prevention 2.1, we argued that the magnitude of the average velocity is not the average speed. Notice the difference when discussing instantaneous values. The magnitude of the instantaneous velocity *is* the instantaneous speed. In an infinitesimal time interval, the magnitude of the displacement is equal to the distance traveled by the particle.

[1] Note that the displacement Δx also approaches zero as Δt approaches zero, so that the ratio looks like 0/0. As Δx and Δt become smaller and smaller, the ratio $\Delta x/\Delta t$ approaches a value equal to the slope of the line tangent to the x-versus-t curve.

[2] As with velocity, we drop the adjective for instantaneous speed: "Speed" means instantaneous speed.

Conceptual Example 2.2 The Velocity of Different Objects

Consider the following one-dimensional motions: **(A)** A ball thrown directly upward rises to a highest point and falls back into the thrower's hand. **(B)** A race car starts from rest and speeds up to 100 m/s. **(C)** A spacecraft drifts through space at constant velocity. Are there any points in the motion of these objects at which the instantaneous velocity has the same value as the average velocity over the entire motion? If so, identify the point(s).

Solution (A) The average velocity for the thrown ball is zero because the ball returns to the starting point; thus its displacement is zero. (Remember that average velocity is defined as $\Delta x/\Delta t$.) There is one point at which the instantaneous velocity is zero—at the top of the motion.

(B) The car's average velocity cannot be evaluated unambiguously with the information given, but it must be some value between 0 and 100 m/s. Because the car will have every instantaneous velocity between 0 and 100 m/s at some time during the interval, there must be some instant at which the instantaneous velocity is equal to the average velocity.

(C) Because the spacecraft's instantaneous velocity is constant, its instantaneous velocity at *any* time and its average velocity over *any* time interval are the same.

Example 2.3 Average and Instantaneous Velocity

A particle moves along the x axis. Its position varies with time according to the expression $x = -4t + 2t^2$ where x is in meters and t is in seconds.[3] The position–time graph for this motion is shown in Figure 2.4. Note that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment $t = 1$ s, and moves in the positive x direction at times $t > 1$ s.

(A) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.

Solution During the first time interval, the slope is negative and hence the average velocity is negative. Thus, we know that the displacement between Ⓐ and Ⓑ must be a negative number having units of meters. Similarly, we expect the displacement between Ⓑ and Ⓓ to be positive.

In the first time interval, we set $t_i = t_A = 0$ and $t_f = t_B = 1$ s. Using Equation 2.1, with $x = -4t + 2t^2$, we obtain for the displacement between $t = 0$ and $t = 1$ s,

$$\begin{aligned}\Delta x_{A\to B} &= x_f - x_i = x_B - x_A \\ &= [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2] \\ &= \boxed{-2\text{ m}}\end{aligned}$$

To calculate the displacement during the second time interval ($t = 1$ s to $t = 3$ s), we set $t_i = t_B = 1$ s and $t_f = t_D = 3$ s:

$$\begin{aligned}\Delta x_{B\to D} &= x_f - x_i = x_D - x_B \\ &= [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2] \\ &= \boxed{+8\text{ m}}\end{aligned}$$

These displacements can also be read directly from the position–time graph.

(B) Calculate the average velocity during these two time intervals.

Solution In the first time interval, $\Delta t = t_f - t_i = t_B - t_A = 1$ s. Therefore, using Equation 2.2 and the displacement calculated in (a), we find that

$$\bar{v}_{x(A\to B)} = \frac{\Delta x_{A\to B}}{\Delta t} = \frac{-2\text{ m}}{1\text{ s}} = \boxed{-2\text{ m/s}}$$

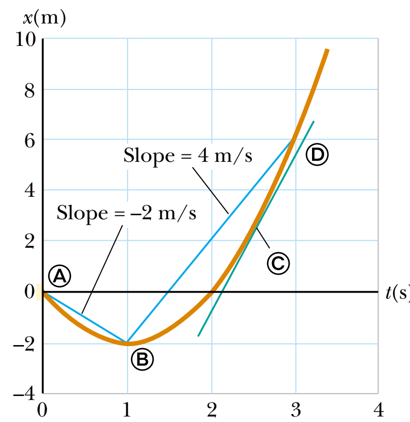


Figure 2.4 (Example 2.3) Position–time graph for a particle having an x coordinate that varies in time according to the expression $x = -4t + 2t^2$.

In the second time interval, $\Delta t = 2$ s; therefore,

$$\bar{v}_{x(B\to D)} = \frac{\Delta x_{B\to D}}{\Delta t} = \frac{8\text{ m}}{2\text{ s}} = \boxed{+4\text{ m/s}}$$

These values are the same as the slopes of the lines joining these points in Figure 2.4.

(C) Find the instantaneous velocity of the particle at $t = 2.5$ s.

Solution We can guess that this instantaneous velocity must be of the same order of magnitude as our previous results, that is, a few meters per second. By measuring the slope of the green line at $t = 2.5$ s in Figure 2.4, we find that

$$v_x = \boxed{+6\text{ m/s}}$$

[3] Simply to make it easier to read, we write the expression as $x = -4t + 2t^2$ rather than as $x = (-4.00\text{ m/s})t + (2.00\text{ m/s}^2)t^{2.00}$. When an equation summarizes measurements, consider its coefficients to have as many significant digits as other data quoted in a problem. Consider its coefficients to have the units required for dimensional consistency. When we start our clocks at $t = 0$, we usually do not mean to limit the precision to a single digit. Consider any zero value in this book to have as many significant figures as you need.

2.3 Acceleration

In the last example, we worked with a situation in which the velocity of a particle changes while the particle is moving. This is an extremely common occurrence. (How constant is your velocity as you ride a city bus or drive on city streets?) It is possible to quantify changes in velocity as a function of time similarly to the way in which we quantify changes in position as a function of time. When the velocity of a particle changes with time, the particle is said to be *accelerating*. For example, the magnitude of the velocity of a car increases when you step on the gas and decreases when you apply the brakes. Let us see how to quantify acceleration.

Suppose an object that can be modeled as a particle moving along the x axis has an initial velocity v_{xi} at time t_i and a final velocity v_{xf} at time t_f, as in Figure 2.5a.

The average acceleration $\bar{a}_x$ of the particle is defined as the *change* in velocity Δv_x divided by the time interval Δt during which that change occurs:

$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \qquad (2.6)$$

Average acceleration

As with velocity, when the motion being analyzed is one-dimensional, we can use positive and negative signs to indicate the direction of the acceleration. Because the dimensions of velocity are L/T and the dimension of time is T, acceleration has dimensions of length divided by time squared, or L/T^2. The SI unit of acceleration is meters per second squared (m/s^2). It might be easier to interpret these units if you think of them as meters per second per second. For example, suppose an object has an acceleration of $+2\ m/s^2$. You should form a mental image of the object having a velocity that is along a straight line and is increasing by 2 m/s during every interval of 1 s. If the object starts from rest, you should be able to picture it moving at a velocity of +2 m/s after 1 s, at +4 m/s after 2 s, and so on.

In some situations, the value of the average acceleration may be different over different time intervals. It is therefore useful to define the *instantaneous acceleration* as the limit of the average acceleration as Δt approaches zero. This concept is analogous to the definition of instantaneous velocity discussed in the previous section. If we imagine that point Ⓐ is brought closer and closer to point Ⓑ in Figure 2.5a and we take the limit of $\Delta v_x/\Delta t$ as Δt approaches zero, we obtain the instantaneous acceleration:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \qquad (2.7)$$

Instantaneous acceleration

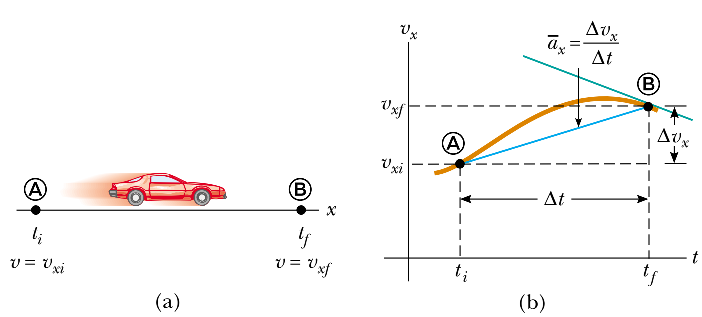


Figure 2.5 (a) A car, modeled as a particle, moving along the x axis from Ⓐ to Ⓑ has velocity v_{xi} at $t = t_i$ and velocity v_{xf} at $t = t_f$. (b) Velocity–time graph (rust) for the particle moving in a straight line. The slope of the blue straight line connecting Ⓐ and Ⓑ is the average acceleration in the time interval $\Delta t = t_f - t_i$.

PITFALL PREVENTION

2.4 Negative Acceleration

Keep in mind that *negative acceleration does not necessarily mean that an object is slowing down.* If the acceleration is negative, and the velocity is negative, the object is speeding up!

PITFALL PREVENTION

2.5 Deceleration

The word *deceleration* has the common popular connotation of *slowing down.* We will not use this word in this text, because it further confuses the definition we have given for negative acceleration.

That is, **the instantaneous acceleration equals the derivative of the velocity with respect to time,** which by definition is the slope of the velocity–time graph. The slope of the green line in Figure 2.5b is equal to the instantaneous acceleration at point Ⓑ. Thus, we see that just as the velocity of a moving particle is the slope at a point on the particle's x-t graph, the acceleration of a particle is the slope at a point on the particle's v_x-t graph. One can interpret the derivative of the velocity with respect to time as the time rate of change of velocity. If a_x is positive, the acceleration is in the positive x direction; if a_x is negative, the acceleration is in the negative x direction.

For the case of motion in a straight line, the direction of the velocity of an object and the direction of its acceleration are related as follows. **When the object's velocity and acceleration are in the same direction, the object is speeding up. On the other hand, when the object's velocity and acceleration are in opposite directions, the object is slowing down.**

To help with this discussion of the signs of velocity and acceleration, we can relate the acceleration of an object to the *force* exerted on the object. In Chapter 5 we formally establish that **force is proportional to acceleration:**

$$F \propto a$$

This proportionality indicates that acceleration is caused by force. Furthermore, force and acceleration are both vectors and the vectors act in the same direction. Thus, let us think about the signs of velocity and acceleration by imagining a force applied to an object and causing it to accelerate. Let us assume that the velocity and acceleration are in the same direction. This situation corresponds to an object moving in some direction that experiences a force acting in the same direction. In this case, the object speeds up! Now suppose the velocity and acceleration are in opposite directions. In this situation, the object moves in some direction and experiences a force acting in the opposite direction. Thus, the object slows down! It is very useful to equate the direction of the acceleration to the direction of a force, because it is easier from our everyday experience to think about what effect a force will have on an object than to think only in terms of the direction of the acceleration.

Quick Quiz 2.2 If a car is traveling eastward and slowing down, what is the direction of the force on the car that causes it to slow down? (a) eastward (b) westward (c) neither of these.

From now on we shall use the term *acceleration* to mean instantaneous acceleration. When we mean average acceleration, we shall always use the adjective *average.*

Because $v_x = dx/dt$, the acceleration can also be written

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \tag{2.8}$$

That is, in one-dimensional motion, the acceleration equals the *second derivative* of x with respect to time.

Figure 2.6 illustrates how an acceleration–time graph is related to a velocity–time graph. The acceleration at any time is the slope of the velocity–time graph at that time. Positive values of acceleration correspond to those points in Figure 2.6a where the velocity is increasing in the positive x direction. The acceleration reaches a maximum at time t_A, when the slope of the velocity–time graph is a maximum. The acceleration then goes to zero at time t_B, when the velocity is a maximum (that is, when the slope of the v_x-t graph is zero). The acceleration is negative when the velocity is decreasing in the positive x direction, and it reaches its most negative value at time t_C.

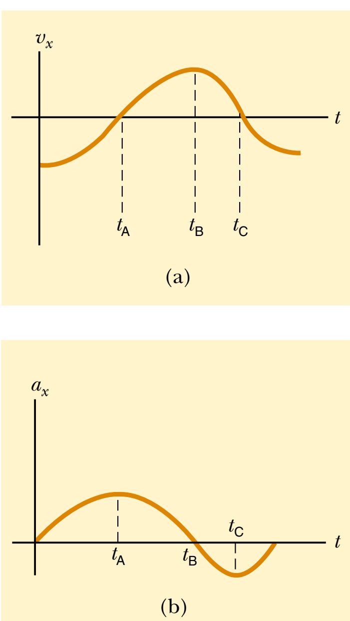


Figure 2.6 The instantaneous acceleration can be obtained from the velocity–time graph (a). At each instant, the acceleration in the a_x versus t graph (b) equals the slope of the line tangent to the v_x versus t curve (a).

Quick Quiz 2.3 Make a velocity–time graph for the car in Figure 2.1a. The speed limit posted on the road sign is 30 km/h. True or false? The car exceeds the speed limit at some time within the interval.

Conceptual Example 2.4 Graphical Relationships between x, v_x, and a_x

The position of an object moving along the x axis varies with time as in Figure 2.7a. Graph the velocity versus time and the acceleration versus time for the object.

Solution The velocity at any instant is the slope of the tangent to the x-t graph at that instant. Between $t = 0$ and $t = t_A$, the slope of the x-t graph increases uniformly, and so the velocity increases linearly, as shown in Figure 2.7b. Between t_A and t_B, the slope of the x-t graph is constant, and so the velocity remains constant. At t_D, the slope of the x-t graph is zero, so the velocity is zero at that instant. Between t_D and t_E, the slope of the x-t graph and thus the velocity are negative and decrease uniformly in this interval. In the interval t_E to t_F, the slope of the x-t graph is still negative, and at t_F it goes to zero. Finally, after t_F, the slope of the x-t graph is zero, meaning that the object is at rest for $t > t_F$.

The acceleration at any instant is the slope of the tangent to the v_x-t graph at that instant. The graph of acceleration versus time for this object is shown in Figure 2.7c. The acceleration is constant and positive between 0 and t_A, where the slope of the v_x-t graph is positive. It is zero between t_A and t_B and for $t > t_F$ because the slope of the v_x-t graph is zero at these times. It is negative between t_B and t_E because the slope of the v_x-t graph is negative during this interval.

Note that the sudden changes in acceleration shown in Figure 2.7c are unphysical. Such instantaneous changes cannot occur in reality.

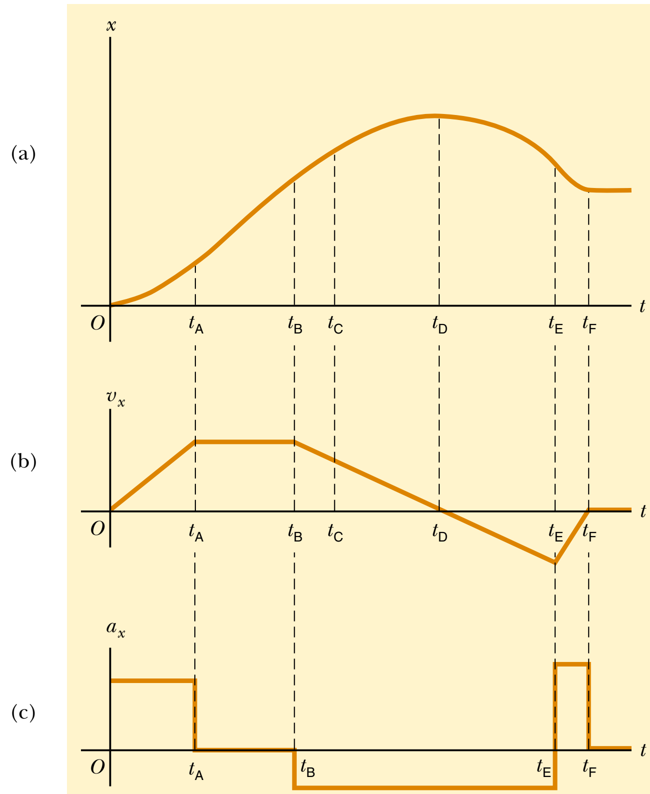


Figure 2.7 (Example 2.4) (a) Position–time graph for an object moving along the x axis. (b) The velocity–time graph for the object is obtained by measuring the slope of the position–time graph at each instant. (c) The acceleration–time graph for the object is obtained by measuring the slope of the velocity–time graph at each instant.

Example 2.5 Average and Instantaneous Acceleration

The velocity of a particle moving along the x axis varies in time according to the expression $v_x = (40 - 5t^2)$ m/s, where t is in seconds.

(A) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

Solution Figure 2.8 is a v_x-t graph that was created from the velocity versus time expression given in the problem statement. Because the slope of the entire v_x-t curve is negative, we expect the acceleration to be negative.

We find the velocities at $t_i = t_A = 0$ and $t_f = t_B = 2.0$ s by substituting these values of t into the expression for the velocity:

$$v_{xA} = (40 - 5t_A{}^2) \text{ m/s} = [40 - 5(0)^2] \text{ m/s} = +40 \text{ m/s}$$

$$v_{xB} = (40 - 5t_B{}^2) \text{ m/s} = [40 - 5(2.0)^2] \text{ m/s} = +20 \text{ m/s}$$

Therefore, the average acceleration in the specified time interval $\Delta t = t_B - t_A = 2.0$ s is

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{xB} - v_{xA}}{t_B - t_A} = \frac{(20 - 40) \text{ m/s}}{(2.0 - 0) \text{ s}}$$

$$= \boxed{-10 \text{ m/s}^2}$$

The negative sign is consistent with our expectations—namely, that the average acceleration, which is represented by the slope of the line joining the initial and final points on the velocity–time graph, is negative.

(B) Determine the acceleration at $t = 2.0$ s.

Solution The velocity at any time t is $v_{xi} = (40 - 5t^2)$ m/s and the velocity at any later time $t + \Delta t$ is

$$v_{xf} = 40 - 5(t + \Delta t)^2 = 40 - 5t^2 - 10t\,\Delta t - 5(\Delta t)^2$$

Therefore, the change in velocity over the time interval Δt is

$$\Delta v_x = v_{xf} - v_{xi} = [-10t\,\Delta t - 5(\Delta t)^2] \text{ m/s}$$

Dividing this expression by Δt and taking the limit of the result as Δt approaches zero gives the acceleration at *any* time t:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \lim_{\Delta t \to 0} (-10t - 5\Delta t) = -10t \text{ m/s}^2$$

Therefore, at $t = 2.0$ s,

$$a_x = (-10)(2.0) \text{ m/s}^2 = \boxed{-20 \text{ m/s}^2}$$

Because the velocity of the particle is positive and the acceleration is negative, the particle is slowing down.

Note that the answers to parts (A) and (B) are different. The average acceleration in (A) is the slope of the blue line in Figure 2.8 connecting points Ⓐ and Ⓑ. The instantaneous acceleration in (B) is the slope of the green line tangent to the curve at point Ⓑ. Note also that the acceleration is *not* constant in this example. Situations involving constant acceleration are treated in Section 2.5.

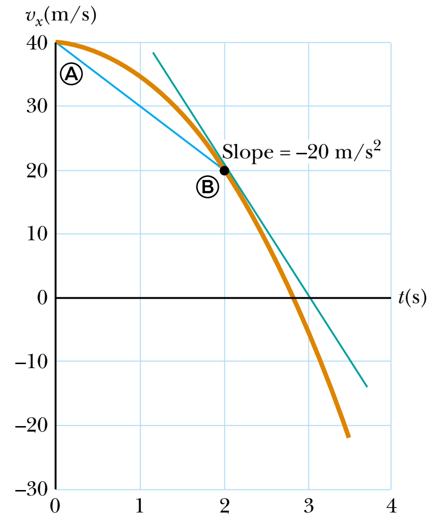


Figure 2.8 (Example 2.5) The velocity–time graph for a particle moving along the x axis according to the expression $v_x = (40 - 5t^2)$ m/s. The acceleration at $t = 2$ s is equal to the slope of the green tangent line at that time.

So far we have evaluated the derivatives of a function by starting with the definition of the function and then taking the limit of a specific ratio. If you are familiar with calculus, you should recognize that there are specific rules for taking derivatives. These rules, which are listed in Appendix B.6, enable us to evaluate derivatives quickly. For instance, one rule tells us that the derivative of any constant is zero. As another example, suppose x is proportional to some power of t, such as in the expression

$$x = At^n$$

where A and n are constants. (This is a very common functional form.) The derivative of x with respect to t is

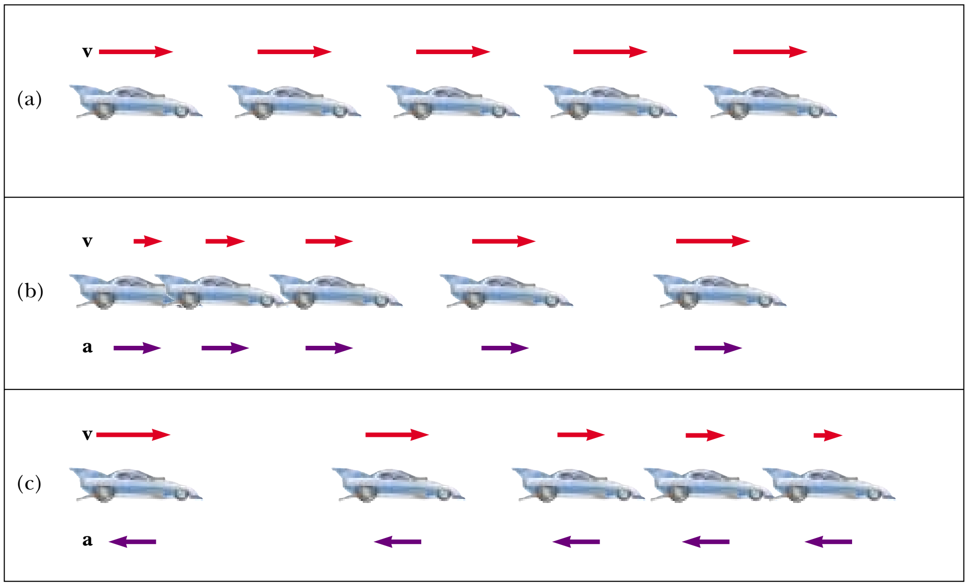
$$\frac{dx}{dt} = nAt^{n-1}$$

Applying this rule to Example 2.5, in which $v_x = 40 - 5t^2$, we find that the acceleration is $a_x = dv_x/dt = -10t$.

2.4 Motion Diagrams

The concepts of velocity and acceleration are often confused with each other, but in fact they are quite different quantities. It is instructive to use motion diagrams to describe the velocity and acceleration while an object is in motion.

A *stroboscopic* photograph of a moving object shows several images of the object, taken as the strobe light flashes at a constant rate. Figure 2.9 represents three sets of strobe photographs of cars moving along a straight roadway in a single direction, from left to right. The time intervals between flashes of the stroboscope are equal in each part of the diagram. In order not to confuse the two vector quantities, we use red for velocity vectors and violet for acceleration vectors in Figure 2.9. The vectors are



Active Figure 2.9 (a) Motion diagram for a car moving at constant velocity (zero acceleration). (b) Motion diagram for a car whose constant acceleration is in the direction of its velocity. The velocity vector at each instant is indicated by a red arrow, and the constant acceleration by a violet arrow. (c) Motion diagram for a car whose constant acceleration is in the direction *opposite* the velocity at each instant.

At the Active Figures link at http://www.pse6.com, you can select the constant acceleration and initial velocity of the car and observe pictorial and graphical representations of its motion.

sketched at several instants during the motion of the object. Let us describe the motion of the car in each diagram.

In Figure 2.9a, the images of the car are equally spaced, showing us that the car moves through the same displacement in each time interval. This is consistent with the car moving with *constant positive velocity* and *zero acceleration*. We could model the car as a particle and describe it as a particle moving with constant velocity.

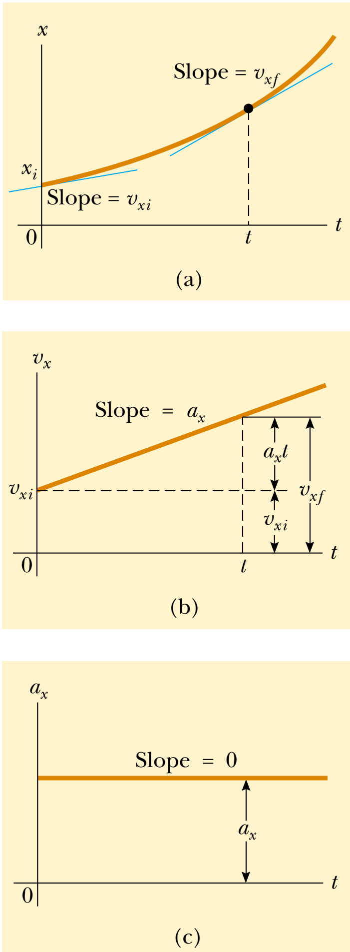
In Figure 2.9b, the images become farther apart as time progresses. In this case, the velocity vector increases in time because the car's displacement between adjacent positions increases in time. This suggests that the car is moving with a *positive velocity* and a *positive acceleration*. The velocity and acceleration are in the same direction. In terms of our earlier force discussion, imagine a force pulling on the car in the same direction it is moving—it speeds up.

In Figure 2.9c, we can tell that the car slows as it moves to the right because its displacement between adjacent images decreases with time. In this case, this suggests that the car moves to the right with a constant negative acceleration. The velocity vector decreases in time and eventually reaches zero. From this diagram we see that the acceleration and velocity vectors are *not* in the same direction. The car is moving with a *positive velocity* but with a *negative acceleration*. (This type of motion is exhibited by a car that skids to a stop after applying its brakes.) The velocity and acceleration are in opposite directions. In terms of our earlier force discussion, imagine a force pulling on the car opposite to the direction it is moving—it slows down.

The violet acceleration vectors in Figures 2.9b and 2.9c are all of the same length. Thus, these diagrams represent motion with constant acceleration. This is an important type of motion that will be discussed in the next section.

Quick Quiz 2.4 Which of the following is true? (a) If a car is traveling eastward, its acceleration is eastward. (b) If a car is slowing down, its acceleration must be negative. (c) A particle with constant acceleration can never stop and stay stopped.

2.5 One-Dimensional Motion with Constant Acceleration



Active Figure 2.10 A particle moving along the x axis with constant acceleration a_x; (a) the position–time graph, (b) the velocity–time graph, and (c) the acceleration–time graph.

At the Active Figures link at http://www.pse6.com, you can adjust the constant acceleration and observe the effect on the position and velocity graphs.

If the acceleration of a particle varies in time, its motion can be complex and difficult to analyze. However, a very common and simple type of one-dimensional motion is that in which the acceleration is constant. When this is the case, the average acceleration $\overline{a}_x$ over any time interval is numerically equal to the instantaneous acceleration a_x at any instant within the interval, and the velocity changes at the same rate throughout the motion.

If we replace $\overline{a}_x$ by a_x in Equation 2.6 and take $t_i = 0$ and t_f to be any later time t, we find that

$$a_x = \frac{v_{xf} - v_{xi}}{t - 0}$$

or

$$v_{xf} = v_{xi} + a_x t \qquad \text{(for constant } a_x\text{)} \tag{2.9}$$

This powerful expression enables us to determine an object's velocity at *any* time t if we know the object's initial velocity v_{xi} and its (constant) acceleration a_x. A velocity–time graph for this constant-acceleration motion is shown in Figure 2.10b. The graph is a straight line, the (constant) slope of which is the acceleration a_x; this is consistent with the fact that $a_x = dv_x/dt$ is a constant. Note that the slope is positive; this indicates a positive acceleration. If the acceleration were negative, then the slope of the line in Figure 2.10b would be negative.

When the acceleration is constant, the graph of acceleration versus time (Fig. 2.10c) is a straight line having a slope of zero.

Because velocity at constant acceleration varies linearly in time according to Equation 2.9, we can express the average velocity in any time interval as the arithmetic mean of the initial velocity v_{xi} and the final velocity v_{xf}:

$$\overline{v}_x = \frac{v_{xi} + v_{xf}}{2} \qquad \text{(for constant } a_x\text{)} \tag{2.10}$$

Note that this expression for average velocity applies *only* in situations in which the acceleration is constant.

We can now use Equations 2.1, 2.2, and 2.10 to obtain the position of an object as a function of time. Recalling that Δx in Equation 2.2 represents $x_f - x_i$, and recognizing that $\Delta t = t_f - t_i = t - 0 = t$, we find

$$x_f - x_i = \overline{v}t = \tfrac{1}{2}(v_{xi} + v_{xf})t$$

Position as a function of velocity and time

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t \qquad \text{(for constant } a_x\text{)} \tag{2.11}$$

This equation provides the final position of the particle at time t in terms of the initial and final velocities.

We can obtain another useful expression for the position of a particle moving with constant acceleration by substituting Equation 2.9 into Equation 2.11:

$$x_f = x_i + \tfrac{1}{2}[v_{xi} + (v_{xi} + a_x t)]t$$

Position as a function of time

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \qquad \text{(for constant } a_x\text{)} \tag{2.12}$$

This equation provides the final position of the particle at time t in terms of the initial velocity and the acceleration.

The position–time graph for motion at constant (positive) acceleration shown in Figure 2.10a is obtained from Equation 2.12. Note that the curve is a parabola.

The slope of the tangent line to this curve at $t = 0$ equals the initial velocity v_{xi}, and the slope of the tangent line at any later time t equals the velocity v_{xf} at that time.

Finally, we can obtain an expression for the final velocity that does not contain time as a variable by substituting the value of t from Equation 2.9 into Equation 2.11:

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})\left(\frac{v_{xf} - v_{xi}}{a_x}\right) = \frac{{v_{xf}}^2 - {v_{xi}}^2}{2a_x}$$

$$\boxed{{v_{xf}}^2 = {v_{xi}}^2 + 2a_x(x_f - x_i) \qquad \text{(for constant } a_x)} \qquad (2.13)$$

Velocity as a function of position

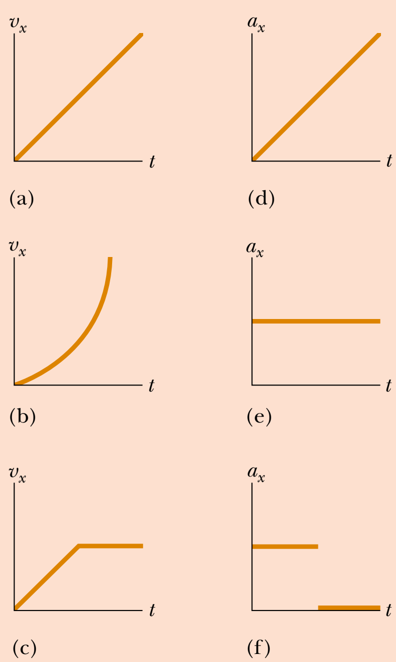
This equation provides the final velocity in terms of the acceleration and the displacement of the particle.

For motion at *zero* acceleration, we see from Equations 2.9 and 2.12 that

$$\left.\begin{aligned} v_{xf} &= v_{xi} = v_x \\ x_f &= x_i + v_x t \end{aligned}\right\} \quad \text{when } a_x = 0$$

That is, when the acceleration of a particle is zero, its velocity is constant and its position changes linearly with time.

Quick Quiz 2.5 In Figure 2.11, match each v_x-t graph on the left with the a_x-t graph on the right that best describes the motion.



Active Figure 2.11 (Quick Quiz 2.5) Parts (a), (b), and (c) are v_x-t graphs of objects in one-dimensional motion. The possible accelerations of each object as a function of time are shown in scrambled order in (d), (e), and (f).

At the Active Figures link at http://www.pse6.com, you can practice matching appropriate velocity vs. time graphs and acceleration vs. time graphs.

Equations 2.9 through 2.13 are **kinematic equations that may be used to solve any problem involving one-dimensional motion at constant acceleration.** Keep in mind that these relationships were derived from the definitions of velocity and

Table 2.2

Kinematic Equations for Motion of a Particle Under Constant Acceleration

Equation	Information Given by Equation
$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

Note: Motion is along the x axis.

acceleration, together with some simple algebraic manipulations and the requirement that the acceleration be constant.

The four kinematic equations used most often are listed in Table 2.2 for convenience. The choice of which equation you use in a given situation depends on what you know beforehand. Sometimes it is necessary to use two of these equations to solve for two unknowns. For example, suppose initial velocity v_{xi} and acceleration a_x are given. You can then find (1) the velocity at time t, using $v_{xf} = v_{xi} + a_x t$ and (2) the position at time t, using $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$. You should recognize that the quantities that vary during the motion are position, velocity, and time.

You will gain a great deal of experience in the use of these equations by solving a number of exercises and problems. Many times you will discover that more than one method can be used to obtain a solution. Remember that these equations of kinematics *cannot* be used in a situation in which the acceleration varies with time. They can be used only when the acceleration is constant.

Example 2.6 Entering the Traffic Flow

(A) Estimate your average acceleration as you drive up the entrance ramp to an interstate highway.

Solution This problem involves more than our usual amount of estimating! We are trying to come up with a value of a_x, but that value is hard to guess directly. The other variables involved in kinematics are position, velocity, and time. Velocity is probably the easiest one to approximate. Let us assume a final velocity of 100 km/h, so that you can merge with traffic. We multiply this value by (1 000 m/1 km) to convert kilometers to meters and then multiply by (1 h/3 600 s) to convert hours to seconds. These two calculations together are roughly equivalent to dividing by 3. In fact, let us just say that the final velocity is $v_{xf} \approx 30$ m/s. (Remember, this type of approximation and the dropping of digits when performing estimations is okay. If you were starting with U.S. customary units, you could approximate 1 mi/h as roughly 0.5 m/s and continue from there.)

Now we assume that you started up the ramp at about one third your final velocity, so that $v_{xi} \approx 10$ m/s. Finally, we assume that it takes about 10 s to accelerate from v_{xi} to v_{xf}, basing this guess on our previous experience in automobiles. We can then find the average acceleration, using Equation 2.6:

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t} \approx \frac{30\text{ m/s} - 10\text{ m/s}}{10\text{ s}}$$

$$= \boxed{2\text{ m/s}^2}$$

Granted, we made many approximations along the way, but **this type of mental effort can be surprisingly useful and often yields results that are not too different from those derived from careful measurements.** Do not be afraid to attempt making educated guesses and doing some fairly drastic number rounding to simplify estimations. Physicists engage in this type of thought analysis all the time.

(B) How far did you go during the first half of the time interval during which you accelerated?

Solution Let us assume that the acceleration is constant, with the value calculated in part (A). Because the motion takes place in a straight line and the velocity is always in the same direction, the distance traveled from the starting point is equal to the final position of the car. We can calculate the final position at 5 s from Equation 2.12:

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$\approx 0 + (10\text{ m/s})(5\text{ s}) + \frac{1}{2}(2\text{ m/s}^2)(5\text{ s})^2 = 50\text{ m} + 25\text{ m}$$

$$= \boxed{75\text{ m}}$$

This result indicates that if you had not accelerated, your initial velocity of 10 m/s would have resulted in a 50-m movement up the ramp during the first 5 s. The additional 25 m is the result of your increasing velocity during that interval.

Example 2.7 Carrier Landing

A jet lands on an aircraft carrier at 140 mi/h ($\approx$63 m/s).

(A) What is its acceleration (assumed constant) if it stops in 2.0 s due to an arresting cable that snags the airplane and brings it to a stop?

Solution We define our x axis as the direction of motion of the jet. A careful reading of the problem reveals that in addition to being given the initial speed of 63 m/s, we also know that the final speed is zero. We also note that we have no information about the change in position of the jet while it is slowing down. Equation 2.9 is the only equation in Table 2.2 that does not involve position, and so we use it to find the acceleration of the jet, modeled as a particle:

$$a_x = \frac{v_{xf} - v_{xi}}{t} \approx \frac{0 - 63\text{ m/s}}{2.0\text{ s}}$$

$$= \boxed{-31\text{ m/s}^2}$$

(B) If the plane touches down at position $x_i = 0$, what is the final position of the plane?

Solution We can now use any of the other three equations in Table 2.2 to solve for the final position. Let us choose Equation 2.11:

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t = 0 + \tfrac{1}{2}(63\text{ m/s} + 0)(2.0\text{ s})$$

$$= \boxed{63\text{ m}}$$

If the plane travels much farther than this, it might fall into the ocean. The idea of using arresting cables to slow down landing aircraft and enable them to land safely on ships originated at about the time of the first World War. The cables are still a vital part of the operation of modern aircraft carriers.

What If? **Suppose the plane lands on the deck of the aircraft carrier with a speed higher than 63 m/s but with the same acceleration as that calculated in part (A). How will that change the answer to part (B)?**

Answer If the plane is traveling faster at the beginning, it will stop farther away from its starting point, so the answer to part (B) should be larger. Mathematically, we see in Equation 2.11 that if v_{xi} is larger, then x_f will be larger.

If the landing deck has a length of 75 m, we can find the maximum initial speed with which the plane can land and still come to rest on the deck at the given acceleration from Equation 2.13:

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

$$\rightarrow v_{xi} = \sqrt{v_{xf}^2 - 2a_x(x_f - x_i)}$$

$$= \sqrt{0 - 2(-31\text{ m/s}^2)(75\text{ m} - 0)}$$

$$= 68\text{ m/s}$$

Example 2.8 Watch Out for the Speed Limit! **Interactive**

A car traveling at a constant speed of 45.0 m/s passes a trooper hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch it, accelerating at a constant rate of 3.00 m/s^2. How long does it take her to overtake the car?

Solution Let us model the car and the trooper as particles. A sketch (Fig. 2.12) helps clarify the sequence of events.

First, we write expressions for the position of each vehicle as a function of time. It is convenient to choose the position of the billboard as the origin and to set $t_B = 0$ as the time the trooper begins moving. At that instant, the car has already traveled a distance of 45.0 m because it has traveled at a constant speed of $v_x = 45.0$ m/s for 1 s. Thus, the initial position of the speeding car is $x_B = 45.0$ m.

Because the car moves with constant speed, its acceleration is zero. Applying Equation 2.12 (with $a_x = 0$) gives for the car's position at any time t:

$$x_{\text{car}} = x_B + v_{x\,\text{car}}t = 45.0\text{ m} + (45.0\text{ m/s})t$$

A quick check shows that at $t = 0$, this expression gives the car's correct initial position when the trooper begins to move: $x_{\text{car}} = x_B = 45.0$ m.

The trooper starts from rest at $t_B = 0$ and accelerates at 3.00 m/s^2 away from the origin. Hence, her position at any time t can be found from Equation 2.12:

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2$$

$$x_{\text{trooper}} = 0 + (0)t + \tfrac{1}{2}a_x t^2 = \tfrac{1}{2}(3.00\text{ m/s}^2)t^2$$

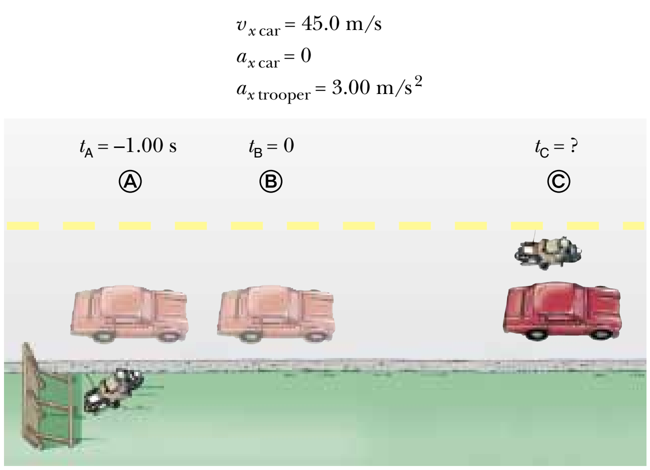


Figure 2.12 (Example 2.8) A speeding car passes a hidden trooper.

The trooper overtakes the car at the instant her position matches that of the car, which is position ©:

$$x_{\text{trooper}} = x_{\text{car}}$$

$$\tfrac{1}{2}(3.00\text{ m/s}^2)t^2 = 45.0\text{ m} + (45.0\text{ m/s})t$$

This gives the quadratic equation

$$1.50t^2 - 45.0t - 45.0 = 0$$

The positive solution of this equation is $t =$ 31.0 s.

(For help in solving quadratic equations, see Appendix B.2.)

What If? **What if the trooper had a more powerful motorcycle with a larger acceleration? How would that change the time at which the trooper catches the car?**

Answer If the motorcycle has a larger acceleration, the trooper will catch up to the car sooner, so the answer for the time will be less than 31 s. Mathematically, let us cast the final quadratic equation above in terms of the parameters in the problem:

$$\tfrac{1}{2}a_x t^2 - v_{x\,\text{car}}t - x_{\text{B}} = 0$$

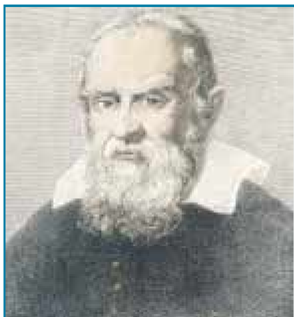
The solution to this quadratic equation is,

$$t = \frac{v_{x\,\text{car}} \pm \sqrt{v_{x\,\text{car}}^2 + 2a_x x_{\text{B}}}}{a_x}$$

$$= \frac{v_{x\,\text{car}}}{a_x} + \sqrt{\frac{v_{x\,\text{car}}^2}{a_x^{\,2}} + \frac{2x_{\text{B}}}{a_x}}$$

where we have chosen the positive sign because that is the only choice consistent with a time $t > 0$. Because all terms on the right side of the equation have the acceleration a_x in the denominator, increasing the acceleration will decrease the time at which the trooper catches the car.

You can study the motion of the car and trooper for various velocities of the car at the Interactive Worked Example link at http://www.pse6.com.



Galileo Galilei

Italian physicist and astronomer (1564–1642)

Galileo formulated the laws that govern the motion of objects in free fall and made many other significant discoveries in physics and astronomy. Galileo publicly defended Nicholaus Copernicus's assertion that the Sun is at the center of the Universe (the heliocentric system). He published *Dialogue Concerning Two New World Systems* to support the Copernican model, a view which the Church declared to be heretical. *(North Wind)*

PITFALL PREVENTION

2.6 *g* and g

Be sure not to confuse the italicized symbol *g* for free-fall acceleration with the nonitalicized symbol g used as the abbreviation for "gram."

2.6 Freely Falling Objects

It is well known that, in the absence of air resistance, all objects dropped near the Earth's surface fall toward the Earth with the same constant acceleration under the influence of the Earth's gravity. It was not until about 1600 that this conclusion was accepted. Before that time, the teachings of the great philosopher Aristotle (384–322 B.C.) had held that heavier objects fall faster than lighter ones.

The Italian Galileo Galilei (1564–1642) originated our present-day ideas concerning falling objects. There is a legend that he demonstrated the behavior of falling objects by observing that two different weights dropped simultaneously from the Leaning Tower of Pisa hit the ground at approximately the same time. Although there is some doubt that he carried out this particular experiment, it is well established that Galileo performed many experiments on objects moving on inclined planes. In his experiments he rolled balls down a slight incline and measured the distances they covered in successive time intervals. The purpose of the incline was to reduce the acceleration; with the acceleration reduced, Galileo was able to make accurate measurements of the time intervals. By gradually increasing the slope of the incline, he was finally able to draw conclusions about freely falling objects because a freely falling ball is equivalent to a ball moving down a vertical incline.

You might want to try the following experiment. Simultaneously drop a coin and a crumpled-up piece of paper from the same height. If the effects of air resistance are negligible, both will have the same motion and will hit the floor at the same time. In the idealized case, in which air resistance is absent, such motion is referred to as *free-fall.* If this same experiment could be conducted in a vacuum, in which air resistance is truly negligible, the paper and coin would fall with the same acceleration even when the paper is not crumpled. On August 2, 1971, such a demonstration was conducted on the Moon by astronaut David Scott. He simultaneously released a hammer and a feather, and they fell together to the lunar surface. This demonstration surely would have pleased Galileo!

When we use the expression *freely falling object,* we do not necessarily refer to an object dropped from rest. **A freely falling object is any object moving freely under the influence of gravity alone, regardless of its initial motion. Objects thrown upward or downward and those released from rest are all falling freely once they**

are released. Any freely falling object experiences an acceleration directed *downward*, regardless of its initial motion.

We shall denote the magnitude of the *free-fall acceleration* by the symbol g. The value of g near the Earth's surface decreases with increasing altitude. Furthermore, slight variations in g occur with changes in latitude. It is common to define "up" as the $+y$ direction and to use y as the position variable in the kinematic equations. At the Earth's surface, the value of g is approximately 9.80 m/s^2. Unless stated otherwise, we shall use this value for g when performing calculations. For making quick estimates, use $g = 10$ m/s^2.

If we neglect air resistance and assume that the free-fall acceleration does not vary with altitude over short vertical distances, then the motion of a freely falling object moving vertically is equivalent to motion in one dimension under constant acceleration. Therefore, the equations developed in Section 2.5 for objects moving with constant acceleration can be applied. The only modification that we need to make in these equations for freely falling objects is to note that the motion is in the vertical direction (the y direction) rather than in the horizontal direction (x) and that the acceleration is downward and has a magnitude of 9.80 m/s^2. Thus, we always choose $a_y = -g = -9.80$ m/s^2, where the negative sign means that the acceleration of a freely falling object is downward. In Chapter 13 we shall study how to deal with variations in g with altitude.

PITFALL PREVENTION

2.7 The Sign of *g*

Keep in mind that g is a *positive number*—it is tempting to substitute -9.80 m/s^2 for g, but resist the temptation. Downward gravitational acceleration is indicated explicitly by stating the acceleration as $a_y = -g$.

PITFALL PREVENTION

2.8 Acceleration at the Top of The Motion

It is a common misconception that the acceleration of a projectile at the top of its trajectory is zero. While the velocity at the top of the motion of an object thrown upward momentarily goes to zero, *the acceleration is still that due to gravity* at this point. If the velocity and acceleration were both zero, the projectile would stay at the top!

Quick Quiz 2.6 A ball is thrown upward. While the ball is in free fall, does its acceleration (a) increase (b) decrease (c) increase and then decrease (d) decrease and then increase (e) remain constant?

Quick Quiz 2.7 After a ball is thrown upward and is in the air, its speed (a) increases (b) decreases (c) increases and then decreases (d) decreases and then increases (e) remains the same.

Conceptual Example 2.9 The Daring Sky Divers

A sky diver jumps out of a hovering helicopter. A few seconds later, another sky diver jumps out, and they both fall along the same vertical line. Ignore air resistance, so that both sky divers fall with the same acceleration. Does the difference in their speeds stay the same throughout the fall? Does the vertical distance between them stay the same throughout the fall?

Solution At any given instant, the speeds of the divers are different because one had a head start. In any time interval Δt after this instant, however, the two divers increase their speeds by the same amount because they have the same acceleration. Thus, the difference in their speeds remains the same throughout the fall.

The first jumper always has a greater speed than the second. Thus, in a given time interval, the first diver covers a greater distance than the second. Consequently, the separation distance between them increases.

Example 2.10 Describing the Motion of a Tossed Ball

A ball is tossed straight up at 25 m/s. Estimate its velocity at 1-s intervals.

Solution Let us choose the upward direction to be positive. Regardless of whether the ball is moving upward or downward, its vertical velocity changes by approximately -10 m/s for every second it remains in the air. It starts out at 25 m/s. After 1 s has elapsed, it is still moving upward but at 15 m/s because its acceleration is downward (downward acceleration causes its velocity to decrease). After another second, its upward velocity has dropped to 5 m/s. Now comes the tricky part—after another half second, its velocity is zero. The ball has gone as high as it will go. After the last half of this 1-s interval, the ball is moving at -5 m/s. (The negative sign tells us that the ball is now moving in the negative direction, that is, *downward*. Its velocity has changed from $+5$ m/s to -5 m/s during that 1-s interval. The change in velocity is still $-5\text{ m/s} - (+5\text{ m/s}) = -10$ m/s in that second.) It continues downward, and after another 1 s has elapsed, it is falling at a velocity of -15 m/s. Finally, after another 1 s, it has reached its original starting point and is moving downward at -25 m/s.

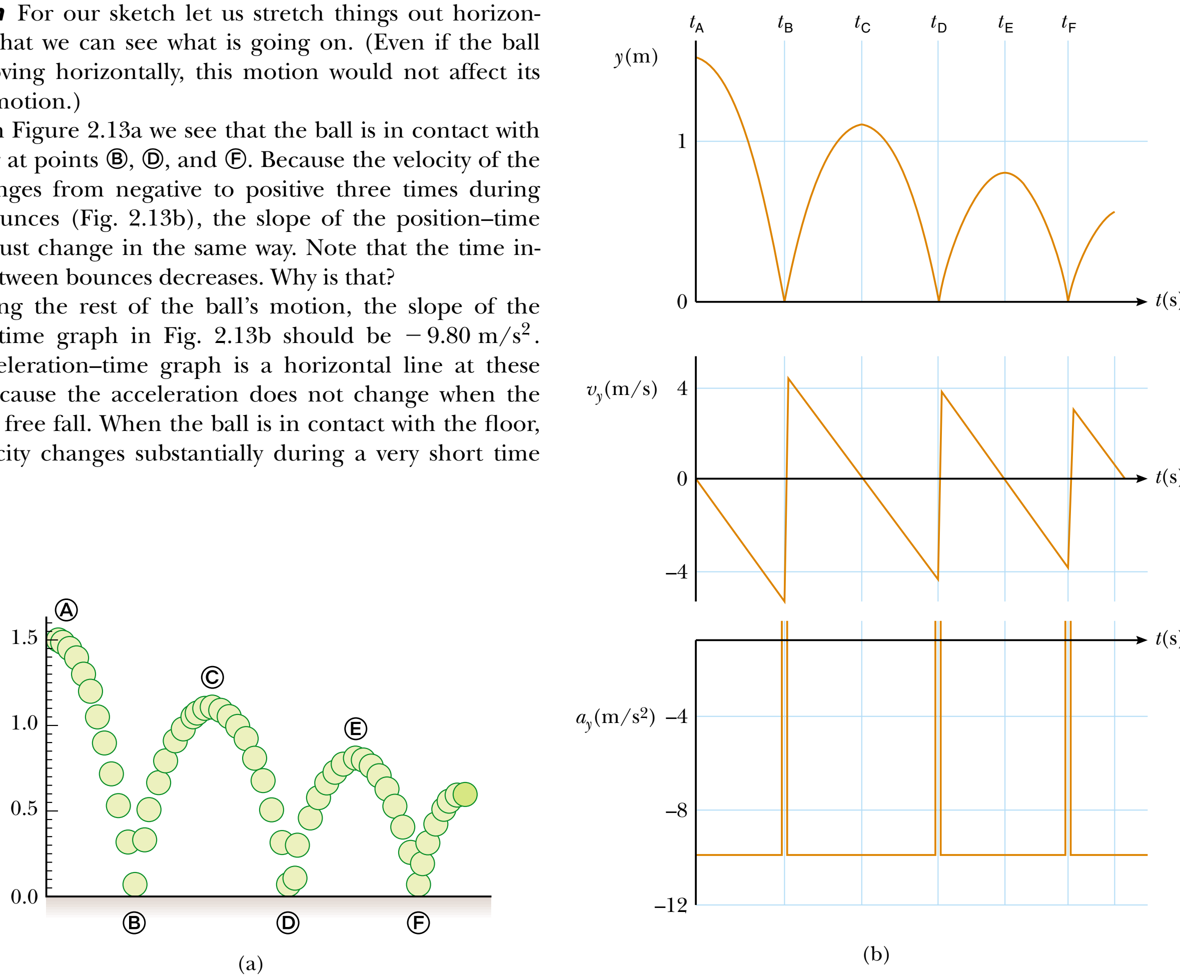
Conceptual Example 2.11 Follow the Bouncing Ball

A tennis ball is dropped from shoulder height (about 1.5 m) and bounces three times before it is caught. Sketch graphs of its position, velocity, and acceleration as functions of time, with the $+y$ direction defined as upward.

Solution For our sketch let us stretch things out horizontally so that we can see what is going on. (Even if the ball were moving horizontally, this motion would not affect its vertical motion.)

From Figure 2.13a we see that the ball is in contact with the floor at points Ⓑ, Ⓓ, and Ⓕ. Because the velocity of the ball changes from negative to positive three times during these bounces (Fig. 2.13b), the slope of the position–time graph must change in the same way. Note that the time interval between bounces decreases. Why is that?

During the rest of the ball's motion, the slope of the velocity–time graph in Fig. 2.13b should be $-9.80\ \text{m/s}^2$. The acceleration–time graph is a horizontal line at these times because the acceleration does not change when the ball is in free fall. When the ball is in contact with the floor, the velocity changes substantially during a very short time interval, and so the acceleration must be quite large and positive. This corresponds to the very steep upward lines on the velocity–time graph and to the spikes on the acceleration–time graph.



Active Figure 2.13 (Conceptual Example 2.11) (a) A ball is dropped from a height of 1.5 m and bounces from the floor. (The horizontal motion is not considered here because it does not affect the vertical motion.) (b) Graphs of position, velocity, and acceleration versus time.

At the Active Figures link at* http://www.pse6.com*, you can adjust both the value for g and the amount of "bounce" of the ball, and observe the resulting motion of the ball both pictorially and graphically.

Quick Quiz 2.8 Which values represent the ball's vertical velocity and acceleration at points Ⓐ, Ⓒ, and Ⓔ in Figure 2.13a?

(a) $v_y = 0,\ a_y = -9.80\ \text{m/s}^2$

(b) $v_y = 0,\ a_y = 9.80\ \text{m/s}^2$

(c) $v_y = 0,\ a_y = 0$

(d) $v_y = -9.80\ \text{m/s},\ a_y = 0$

Example 2.12 Not a Bad Throw for a Rookie! **Interactive**

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14. Using $t_A = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine **(A)** the time at which the stone reaches its maximum height, **(B)** the maximum height, **(C)** the time at which the stone returns to the height from which it was thrown, **(D)** the velocity of the stone at this instant, and **(E)** the velocity and position of the stone at $t = 5.00$ s.

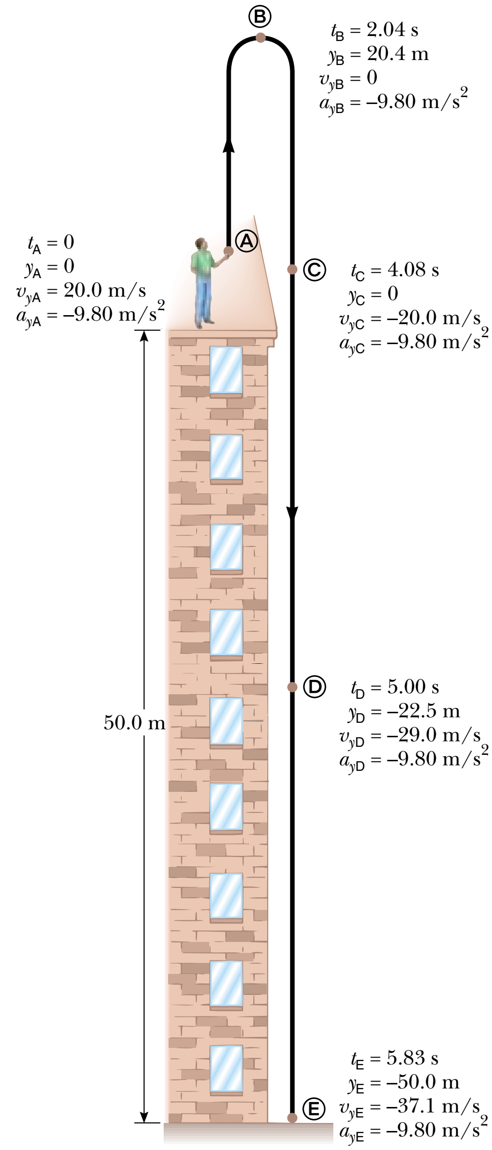


Figure 2.14 (Example 2.12) Position and velocity versus time for a freely falling stone thrown initially upward with a velocity $v_{yi} = 20.0$ m/s.

Solution (A) As the stone travels from Ⓐ to Ⓑ, its velocity must change by 20 m/s because it stops at Ⓑ. Because gravity causes vertical velocities to change by about 10 m/s for every second of free fall, it should take the stone about 2 s to go from Ⓐ to Ⓑ in our drawing. To calculate the exact time t_B at which the stone reaches maximum height, we use Equation 2.9, $v_{yB} = v_{yA} + a_y t$, noting that $v_{yB} = 0$ and setting the start of our clock readings at $t_A = 0$:

$$0 = 20.0\ \text{m/s} + (-9.80\ \text{m/s}^2)t$$

$$t = t_B = \frac{20.0\ \text{m/s}}{9.80\ \text{m/s}^2} = \boxed{2.04\ \text{s}}$$

Our estimate was pretty close.

(B) Because the average velocity for this first interval is 10 m/s (the average of 20 m/s and 0 m/s) and because it travels for about 2 s, we expect the stone to travel about 20 m. By substituting our time into Equation 2.12, we can find the maximum height as measured from the position of the thrower, where we set $y_A = 0$:

$$y_{\text{max}} = y_B = y_A + v_{xA}t + \tfrac{1}{2}a_y t^2$$

$$y_B = 0 + (20.0\ \text{m/s})(2.04\ \text{s}) + \tfrac{1}{2}(-9.80\ \text{m/s}^2)(2.04\ \text{s})^2$$

$$= \boxed{20.4\ \text{m}}$$

Our free-fall estimates are very accurate.

(C) There is no reason to believe that the stone's motion from Ⓑ to Ⓒ is anything other than the reverse of its motion from Ⓐ to Ⓑ. The motion from Ⓐ to Ⓒ is symmetric. Thus, the time needed for it to go from Ⓐ to Ⓒ should be twice the time needed for it to go from Ⓐ to Ⓑ. When the stone is back at the height from which it was thrown (position Ⓒ), the y coordinate is again zero. Using Equation 2.12, with $y_C = 0$, we obtain

$$y_C = y_A + v_{yA}t + \tfrac{1}{2}a_y t^2$$

$$0 = 0 + 20.0t - 4.90t^2$$

This is a quadratic equation and so has two solutions for $t = t_C$. The equation can be factored to give

$$t(20.0 - 4.90t) = 0$$

One solution is $t = 0$, corresponding to the time the stone starts its motion. The other solution is $\boxed{t = 4.08\ \text{s},}$ which is the solution we are after. Notice that it is double the value we calculated for t_B.

(D) Again, we expect everything at Ⓒ to be the same as it is at Ⓐ, except that the velocity is now in the opposite direction. The value for t found in (c) can be inserted into Equation 2.9 to give

$$v_{yC} = v_{yA} + a_y t = 20.0\ \text{m/s} + (-9.80\ \text{m/s}^2)(4.08\ \text{s})$$

$$= \boxed{-20.0\ \text{m/s}}$$

The velocity of the stone when it arrives back at its original height is equal in magnitude to its initial velocity but opposite in direction.

(E) For this part we ignore the first part of the motion (Ⓐ→Ⓑ) and consider what happens as the stone falls from position Ⓑ, where it has zero vertical velocity, to position Ⓓ. We define the initial time as $t_B = 0$. Because the given time for this part of the motion relative to our new zero of time is $5.00\text{ s} - 2.04\text{ s} = 2.96\text{ s}$, we estimate that the acceleration due to gravity will have changed the speed by about 30 m/s. We can calculate this from Equation 2.9, where we take $t = 2.96$ s:

$$v_{yD} = v_{yB} + a_y t = 0\text{ m/s} + (-9.80\text{ m/s}^2)(2.96\text{ s})$$
$$= \boxed{-29.0\text{ m/s}}$$

We could just as easily have made our calculation between positions Ⓐ (where we return to our original initial time $t_A = 0$) and Ⓓ:

$$v_{yD} = v_{yA} + a_y t = 20.0\text{ m/s} + (-9.80\text{ m/s}^2)(5.00\text{ s})$$
$$= -29.0\text{ m/s}$$

To further demonstrate that we can choose different initial instants of time, let us use Equation 2.12 to find the position of the stone at $t_D = 5.00$ s (with respect to $t_A = 0$) by defining a new initial instant, $t_C = 0$:

$$y_D = y_C + v_{yC}t + \tfrac{1}{2}a_y t^2$$
$$= 0 + (-20.0\text{ m/s})(5.00\text{ s} - 4.08\text{ s})$$
$$+ \tfrac{1}{2}(-9.80\text{ m/s}^2)(5.00\text{ s} - 4.08\text{ s})^2$$
$$= \boxed{-22.5\text{ m}}$$

What If? **What if the building were 30.0 m tall instead of 50.0 m tall? Which answers in parts (A) to (E) would change?**

Answer None of the answers would change. All of the motion takes place in the air, and the stone does not interact with the ground during the first 5.00 s. (Notice that even for a 30.0-m tall building, the stone is above the ground at $t = 5.00$ s.) Thus, the height of the building is not an issue. Mathematically, if we look back over our calculations, we see that we never entered the height of the building into any equation.

You can study the motion of the thrown ball at the Interactive Worked Example link at http://www.pse6.com.

2.7 Kinematic Equations Derived from Calculus

This section assumes the reader is familiar with the techniques of integral calculus. If you have not yet studied integration in your calculus course, you should skip this section or cover it after you become familiar with integration.

The velocity of a particle moving in a straight line can be obtained if its position as a function of time is known. Mathematically, the velocity equals the derivative of the position with respect to time. It is also possible to find the position of a particle if its velocity is known as a function of time. In calculus, the procedure used to perform this task is referred to either as *integration* or as finding the *antiderivative.* Graphically, it is equivalent to finding the area under a curve.

Suppose the v_x-t graph for a particle moving along the x axis is as shown in Figure 2.15. Let us divide the time interval $t_f - t_i$ into many small intervals, each of

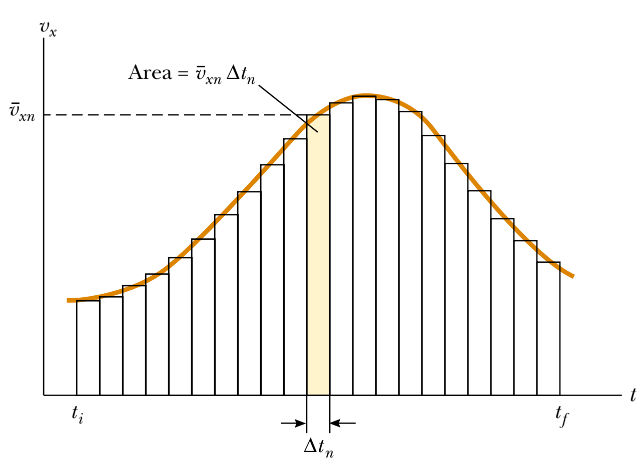


Figure 2.15 Velocity versus time for a particle moving along the x axis. The area of the shaded rectangle is equal to the displacement Δx in the time interval Δt_n, while the total area under the curve is the total displacement of the particle.

duration Δt_n. From the definition of average velocity we see that the displacement during any small interval, such as the one shaded in Figure 2.15, is given by $\Delta x_n = \overline{v}_{xn}\,\Delta t_n$ where $\overline{v}_{xn}$ is the average velocity in that interval. Therefore, the displacement during this small interval is simply the area of the shaded rectangle. The total displacement for the interval $t_f - t_i$ is the sum of the areas of all the rectangles:

$$\Delta x = \sum_n \overline{v}_{xn}\,\Delta t_n$$

where the symbol Σ (upper case Greek sigma) signifies a sum over all terms, that is, over all values of n. In this case, the sum is taken over all the rectangles from t_i to t_f. Now, as the intervals are made smaller and smaller, the number of terms in the sum increases and the sum approaches a value equal to the area under the velocity–time graph. Therefore, in the limit $n \rightarrow \infty$, or $\Delta t_n \rightarrow 0$, the displacement is

$$\Delta x = \lim_{\Delta t_n \rightarrow 0} \sum_n v_{xn}\,\Delta t_n \tag{2.14}$$

or

Displacement = area under the v_x-t graph

Note that we have replaced the average velocity $\overline{v}_{xn}$ with the instantaneous velocity v_{xn} in the sum. As you can see from Figure 2.15, this approximation is valid in the limit of very small intervals. Therefore if we know the v_x-t graph for motion along a straight line, we can obtain the displacement during any time interval by measuring the area under the curve corresponding to that time interval.

The limit of the sum shown in Equation 2.14 is called a **definite integral** and is written

Definite integral

$$\lim_{\Delta t_n \rightarrow 0} \sum_n v_{xn}\,\Delta t_n = \int_{t_i}^{t_f} v_x(t)\,dt \tag{2.15}$$

where $v_x(t)$ denotes the velocity at any time t. If the explicit functional form of $v_x(t)$ is known and the limits are given, then the integral can be evaluated. Sometimes the v_x-t graph for a moving particle has a shape much simpler than that shown in Figure 2.15. For example, suppose a particle moves at a constant velocity v_{xi}. In this case, the v_x-t graph is a horizontal line, as in Figure 2.16, and the displacement of the particle during the time interval Δt is simply the area of the shaded rectangle:

$$\Delta x = v_{xi}\,\Delta t \qquad \text{(when } v_x = v_{xi} = \text{constant)}$$

As another example, consider a particle moving with a velocity that is proportional to t, as in Figure 2.17. Taking $v_x = a_x t$, where a_x is the constant of proportionality (the

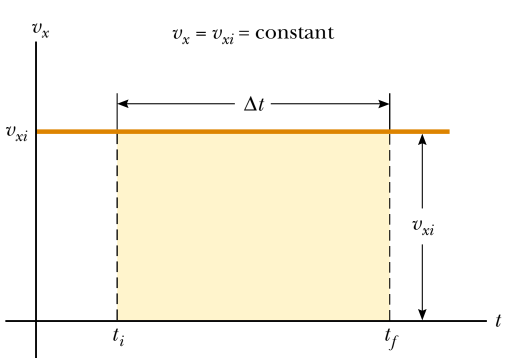


Figure 2.16 The velocity–time curve for a particle moving with constant velocity v_{xi}. The displacement of the particle during the time interval $t_f - t_i$ is equal to the area of the shaded rectangle.

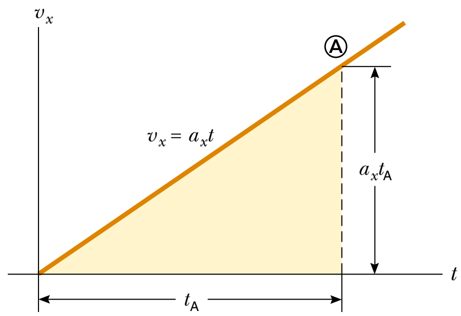


Figure 2.17 The velocity–time curve for a particle moving with a velocity that is proportional to the time.

acceleration), we find that the displacement of the particle during the time interval $t = 0$ to $t = t_A$ is equal to the area of the shaded triangle in Figure 2.17:

$$\Delta x = \tfrac{1}{2}(t_A)(a_x t_A) = \tfrac{1}{2}\, a_x t_A{}^2$$

Kinematic Equations

We now use the defining equations for acceleration and velocity to derive two of our kinematic equations, Equations 2.9 and 2.12.

The defining equation for acceleration (Eq. 2.7),

$$a_x = \frac{dv_x}{dt}$$

may be written as $dv_x = a_x dt$ or, in terms of an integral (or antiderivative), as

$$v_{xf} - v_{xi} = \int_0^t a_x\, dt$$

For the special case in which the acceleration is constant, a_x can be removed from the integral to give

$$v_{xf} - v_{xi} = a_x \int_0^t dt = a_x(t - 0) = a_x t \qquad (2.16)$$

which is Equation 2.9.

Now let us consider the defining equation for velocity (Eq. 2.5):

$$v_x = \frac{dx}{dt}$$

We can write this as $dx = v_x\, dt$, or in integral form as

$$x_f - x_i = \int_0^t v_x\, dt$$

Because $v_x = v_{xf} = v_{xi} + a_x t$, this expression becomes

$$\begin{aligned} x_f - x_i &= \int_0^t (v_{xi} + a_x t)\, dt = \int_0^t v_{xi}\, dt + a_x \int_0^t t\, dt = v_{xi}(t - 0) + a_x\left(\frac{t^2}{2} - 0\right) \\ &= v_{xi} t + \tfrac{1}{2} a_x t^2 \end{aligned}$$

which is Equation 2.12.

Besides what you might expect to learn about physics concepts, a very valuable skill you should hope to take away from your physics course is the ability to solve complicated problems. The way physicists approach complex situations and break them down into manageable pieces is extremely useful. On the next page is a general problem-solving strategy that will help guide you through the steps. To help you remember the steps of the strategy, they are called *Conceptualize, Categorize, Analyze, and Finalize.*

GENERAL PROBLEM-SOLVING STRATEGY

Conceptualize

- The first thing to do when approaching a problem is to *think about* and *understand* the situation. Study carefully any diagrams, graphs, tables, or photographs that accompany the problem. Imagine a movie, running in your mind, of what happens in the problem.
- If a diagram is not provided, you should almost always make a quick drawing of the situation. Indicate any known values, perhaps in a table or directly on your sketch.
- Now focus on what algebraic or numerical information is given in the problem. Carefully read the problem statement, looking for key phrases such as "starts from rest" ($v_i = 0$), "stops" ($v_f = 0$), or "freely falls" ($a_y = -g = -9.80 \text{ m/s}^2$).
- Now focus on the expected result of solving the problem. Exactly what is the question asking? Will the final result be numerical or algebraic? Do you know what units to expect?
- Don't forget to incorporate information from your own experiences and common sense. What should a reasonable answer look like? For example, you wouldn't expect to calculate the speed of an automobile to be 5×10^6 m/s.

Categorize

- Once you have a good idea of what the problem is about, you need to *simplify* the problem. Remove the details that are not important to the solution. For example, model a moving object as a particle. If appropriate, ignore air resistance or friction between a sliding object and a surface.
- Once the problem is simplified, it is important to *categorize* the problem. Is it a simple *plug-in problem,* such that numbers can be simply substituted into a definition? If so, the problem is likely to be finished when this substitution is done. If not, you face what we can call an *analysis problem*—the situation must be analyzed more deeply to reach a solution.
- If it is an analysis problem, it needs to be categorized further. Have you seen this type of problem before? Does it fall into the growing list of types of problems that you have solved previously? Being able to classify a problem can make it much easier to lay out a plan to solve it. For example, if your simplification shows that the problem can be treated as a particle moving under constant acceleration and you have already solved such a problem (such as the examples in Section 2.5), the solution to the present problem follows a similar pattern.

Analyze

- Now you must analyze the problem and strive for a mathematical solution. Because you have already categorized the problem, it should not be too difficult to select relevant equations that apply to the type of situation in the problem. For example, if the problem involves a particle moving under constant acceleration, Equations 2.9 to 2.13 are relevant.
- Use algebra (and calculus, if necessary) to solve symbolically for the unknown variable in terms of what is given. Substitute in the appropriate numbers, calculate the result, and round it to the proper number of significant figures.

Finalize

- This is the most important part. Examine your numerical answer. Does it have the correct units? Does it meet your expectations from your conceptualization of the problem? What about the algebraic form of the result—before you substituted numerical values? Does it make sense? Examine the variables in the problem to see whether the answer would change in a physically meaningful way if they were drastically increased or decreased or even became zero. Looking at limiting cases to see whether they yield expected values is a very useful way to make sure that you are obtaining reasonable results.
- Think about how this problem compares with others you have solved. How was it similar? In what critical ways did it differ? Why was this problem assigned? You should have learned something by doing it. Can you figure out what? If it is a new category of problem, be sure you understand it so that you can use it as a model for solving future problems in the same category.

When solving complex problems, you may need to identify a series of sub-problems and apply the problem-solving strategy to each. For very simple problems, you probably don't need this strategy at all. But when you are looking at a problem and you don't know what to do next, remember the steps in the strategy and use them as a guide.

For practice, it would be useful for you to go back over the examples in this chapter and identify the *Conceptualize, Categorize, Analyze,* and *Finalize* steps. In the next chapter, we will begin to show these steps explicitly in the examples.

SUMMARY

Take a practice test for this chapter by clicking the Practice Test link at **http://www.pse6.com.**

After a particle moves along the x axis from some initial position x_i to some final position x_f, its **displacement** is

$$\Delta x \equiv x_f - x_i \tag{2.1}$$

The **average velocity** of a particle during some time interval is the displacement Δx divided by the time interval Δt during which that displacement occurs:

$$\bar{v}_x \equiv \frac{\Delta x}{\Delta t} \tag{2.2}$$

The **average speed** of a particle is equal to the ratio of the total distance it travels to the total time interval during which it travels that distance:

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}} \tag{2.3}$$

The **instantaneous velocity** of a particle is defined as the limit of the ratio $\Delta x/\Delta t$ as Δt approaches zero. By definition, this limit equals the derivative of x with respect to t, or the time rate of change of the position:

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \tag{2.5}$$

The **instantaneous speed** of a particle is equal to the magnitude of its instantaneous velocity.

The **average acceleration** of a particle is defined as the ratio of the change in its velocity Δv_x divided by the time interval Δt during which that change occurs:

$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \tag{2.6}$$

The **instantaneous acceleration** is equal to the limit of the ratio $\Delta v_x/\Delta t$ as Δt approaches 0. By definition, this limit equals the derivative of v_x with respect to t, or the time rate of change of the velocity:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \tag{2.7}$$

When the object's velocity and acceleration are in the same direction, the object is speeding up. On the other hand, when the object's velocity and acceleration are in opposite directions, the object is slowing down. Remembering that $F \propto a$ is a useful way to identify the direction of the acceleration.

The **equations of kinematics** for a particle moving along the x axis with uniform acceleration a_x (constant in magnitude and direction) are

$$v_{xf} = v_{xi} + a_x t \tag{2.9}$$

$$x_f = x_i + \bar{v}_x t = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t \tag{2.11}$$

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \tag{2.12}$$

$$v_{xf}^{\,2} = v_{xi}^{\,2} + 2a_x(x_f - x_i) \tag{2.13}$$

An object falling freely in the presence of the Earth's gravity experiences a free-fall acceleration directed toward the center of the Earth. If air resistance is neglected, if the motion occurs near the surface of the Earth, and if the range of the motion is small compared with the Earth's radius, then the free-fall acceleration g is constant over the range of motion, where g is equal to 9.80 m/s^2.

Complicated problems are best approached in an organized manner. You should be able to recall and apply the *Conceptualize, Categorize, Analyze,* and *Finalize* steps of the General Problem-Solving Strategy when you need them.

QUESTIONS

1. The speed of sound in air is 331 m/s. During the next thunderstorm, try to estimate your distance from a lightning bolt by measuring the time lag between the flash and the thunderclap. You can ignore the time it takes for the light flash to reach you. Why?

2. The average velocity of a particle moving in one dimension has a positive value. Is it possible for the instantaneous velocity to have been negative at any time in the interval? Suppose the particle started at the origin $x = 0$. If its average velocity is positive, could the particle ever have been in the $-x$ region of the axis?

3. If the average velocity of an object is zero in some time interval, what can you say about the displacement of the object for that interval?

4. Can the instantaneous velocity of an object at an instant of time ever be greater in magnitude than the average velocity over a time interval containing the instant? Can it ever be less?

5. If an object's average velocity is nonzero over some time interval, does this mean that its instantaneous velocity is never zero during the interval? Explain your answer.

6. If an object's average velocity is zero over some time interval, show that its instantaneous velocity must be zero at some time during the interval. It may be useful in your proof to sketch a graph of x versus t and to note that $v_x(t)$ is a continuous function.

7. If the velocity of a particle is nonzero, can its acceleration be zero? Explain.

8. If the velocity of a particle is zero, can its acceleration be nonzero? Explain.

9. Two cars are moving in the same direction in parallel lanes along a highway. At some instant, the velocity of car A exceeds the velocity of car B. Does this mean that the acceleration of A is greater than that of B? Explain.

10. Is it possible for the velocity and the acceleration of an object to have opposite signs? If not, state a proof. If so, give an example of such a situation and sketch a velocity–time graph to prove your point.

11. Consider the following combinations of signs and values for velocity and acceleration of a particle with respect to a one-dimensional x axis:

	Velocity	*Acceleration*
a.	Positive	Positive
b.	Positive	Negative
c.	Positive	Zero
d.	Negative	Positive
e.	Negative	Negative
f.	Negative	Zero
g.	Zero	Positive
h.	Zero	Negative

Describe what a particle is doing in each case, and give a real life example for an automobile on an east-west one-dimensional axis, with east considered the positive direction.

12. Can the equations of kinematics (Eqs. 2.9–2.13) be used in a situation where the acceleration varies in time? Can they be used when the acceleration is zero?

13. A stone is thrown vertically upward from the roof of a building. Does the position of the stone depend on the location chosen for the origin of the coordinate system? Does the stone's velocity depend on the choice of origin? Explain your answers.

14. A child throws a marble into the air with an initial speed v_i. Another child drops a ball at the same instant. Compare the accelerations of the two objects while they are in flight.

15. A student at the top of a building of height h throws one ball upward with a speed of v_i and then throws a second ball downward with the same initial speed, v_i. How do the final velocities of the balls compare when they reach the ground?

16. An object falls freely from height h. It is released at time zero and strikes the ground at time t. (a) When the object is at height $0.5h$, is the time earlier than $0.5t$, equal to $0.5t$, or later than $0.5t$? (b) When the time is $0.5t$, is the height of the object greater than $0.5h$, equal to $0.5h$, or less than $0.5h$? Give reasons for your answers.

17. You drop a ball from a window on an upper floor of a building. It strikes the ground with speed v. You now repeat the drop, but you have a friend down on the street who throws another ball upward at speed v. Your friend throws the ball upward at exactly the same time that you drop yours from the window. At some location, the balls pass each other. Is this location *at* the halfway point between window and ground, *above* this point, or *below* this point?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 2.1 Position, Velocity, and Speed

1. The position of a pinewood derby car was observed at various times; the results are summarized in the following table. Find the average velocity of the car for (a) the first second, (b) the last 3 s, and (c) the entire period of observation.

t(s)	0	1.0	2.0	3.0	4.0	5.0
x(m)	0	2.3	9.2	20.7	36.8	57.5

2. (a) Sand dunes in a desert move over time as sand is swept up the windward side to settle in the lee side. Such "walking" dunes have been known to walk 20 feet in a year and can travel as much as 100 feet per year in particularly windy times. Calculate the average speed in each case in m/s. (b) Fingernails grow at the rate of drifting continents, on the order of 10 mm/yr. Approximately how long did it take for North America to separate from Europe, a distance of about 3 000 mi?

3. The position versus time for a certain particle moving along the x axis is shown in Figure P2.3. Find the average velocity in the time intervals (a) 0 to 2 s, (b) 0 to 4 s, (c) 2 s to 4 s, (d) 4 s to 7 s, (e) 0 to 8 s.

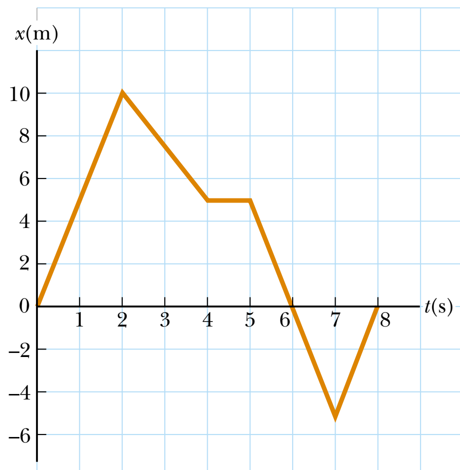


Figure P2.3 Problems 3 and 9

4. A particle moves according to the equation $x = 10t^2$ where x is in meters and t is in seconds. (a) Find the average velocity for the time interval from 2.00 s to 3.00 s. (b) Find the average velocity for the time interval from 2.00 to 2.10 s.

5. A person walks first at a constant speed of 5.00 m/s along a straight line from point A to point B and then back along the line from B to A at a constant speed of 3.00 m/s. What is (a) her average speed over the entire trip? (b) her average velocity over the entire trip?

Section 2.2 Instantaneous Velocity and Speed

6. The position of a particle moving along the x axis varies in time according to the expression $x = 3t^2$, where x is in meters and t is in seconds. Evaluate its position (a) at $t = 3.00$ s and (b) at $3.00\text{ s} + \Delta t$. (c) Evaluate the limit of $\Delta x/\Delta t$ as Δt approaches zero, to find the velocity at $t = 3.00$ s.

7. A position-time graph for a particle moving along the x axis is shown in Figure P2.7. (a) Find the average velocity in the time interval $t = 1.50$ s to $t = 4.00$ s. (b) Determine the instantaneous velocity at $t = 2.00$ s by measuring the slope of the tangent line shown in the graph. (c) At what value of t is the velocity zero?

8. (a) Use the data in Problem 1 to construct a smooth graph of position versus time. (b) By constructing tangents to the $x(t)$ curve, find the instantaneous velocity of the car at several instants. (c) Plot the instantaneous velocity versus time and, from this, determine the average acceleration of the car. (d) What was the initial velocity of the car?

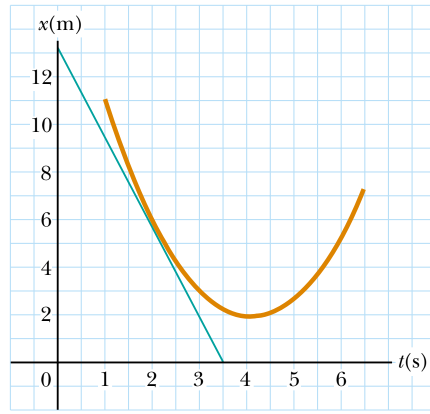


Figure P2.7

9. Find the instantaneous velocity of the particle described in Figure P2.3 at the following times: (a) $t = 1.0$ s, (b) $t = 3.0$ s, (c) $t = 4.5$ s, and (d) $t = 7.5$ s.

10. A hare and a tortoise compete in a race over a course 1.00 km long. The tortoise crawls straight and steadily at its maximum speed of 0.200 m/s toward the finish line. The hare runs at its maximum speed of 8.00 m/s toward the goal for 0.800 km and then stops to tease the tortoise. How close to the goal can the hare let the tortoise approach before resuming the race, which the tortoise wins in a photo finish? Assume that, when moving, both animals move steadily at their respective maximum speeds.

Section 2.3 Acceleration

11. A 50.0-g superball traveling at 25.0 m/s bounces off a brick wall and rebounds at 22.0 m/s. A high-speed camera records this event. If the ball is in contact with the wall for 3.50 ms, what is the magnitude of the average acceleration of the ball during this time interval? (*Note:* 1 ms $= 10^{-3}$ s.)

12. A particle starts from rest and accelerates as shown in Figure P2.12. Determine (a) the particle's speed at $t = 10.0$ s and at $t = 20.0$ s, and (b) the distance traveled in the first 20.0 s.

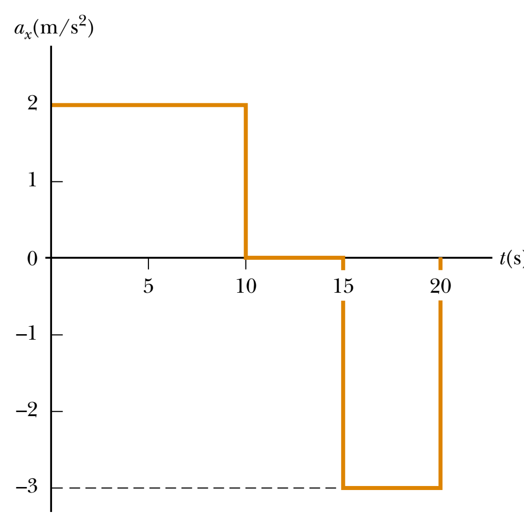


Figure P2.12

13. Secretariat won the Kentucky Derby with times for successive quarter-mile segments of 25.2 s, 24.0 s, 23.8 s, and 23.0 s. (a) Find his average speed during each quarter-mile segment. (b) Assuming that Secretariat's instantaneous speed at the finish line was the same as the average speed during the final quarter mile, find his average acceleration for the entire race. (Horses in the Derby start from rest.)

14. A velocity–time graph for an object moving along the x axis is shown in Figure P2.14. (a) Plot a graph of the acceleration versus time. (b) Determine the average acceleration of the object in the time intervals $t = 5.00$ s to $t = 15.0$ s and $t = 0$ to $t = 20.0$ s.

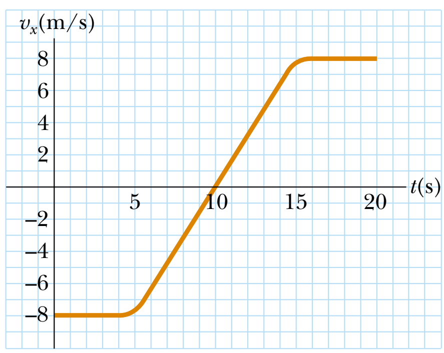


Figure P2.14

$\boxed{15.}$ A particle moves along the x axis according to the equation $x = 2.00 + 3.00t - 1.00t^2$, where x is in meters and t is in seconds. At $t = 3.00$ s, find (a) the position of the particle, (b) its velocity, and (c) its acceleration.

16. An object moves along the x axis according to the equation $x(t) = (3.00t^2 - 2.00t + 3.00)$ m. Determine (a) the average speed between $t = 2.00$ s and $t = 3.00$ s, (b) the instantaneous speed at $t = 2.00$ s and at $t = 3.00$ s, (c) the average acceleration between $t = 2.00$ s and $t = 3.00$ s, and (d) the instantaneous acceleration at $t = 2.00$ s and $t = 3.00$ s.

17. Figure P2.17 shows a graph of v_x versus t for the motion of a motorcyclist as he starts from rest and moves along the road in a straight line. (a) Find the average acceleration for the time interval $t = 0$ to $t = 6.00$ s. (b) Estimate the time at which the acceleration has its greatest positive value and the value of the acceleration at that instant. (c) When is the acceleration zero? (d) Estimate the maximum negative value of the acceleration and the time at which it occurs.

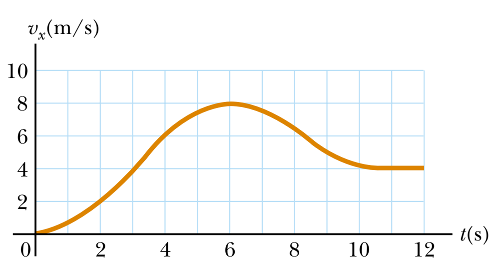


Figure P2.17

Section 2.4 Motion Diagrams

18. Draw motion diagrams for (a) an object moving to the right at constant speed, (b) an object moving to the right and speeding up at a constant rate, (c) an object moving to the right and slowing down at a constant rate, (d) an object moving to the left and speeding up at a constant rate, and (e) an object moving to the left and slowing down at a constant rate. (f) How would your drawings change if the changes in speed were not uniform; that is, if the speed were not changing at a constant rate?

Section 2.5 One-Dimensional Motion with Constant Acceleration

19. Jules Verne in 1865 suggested sending people to the Moon by firing a space capsule from a 220-m-long cannon with a launch speed of 10.97 km/s. What would have been the unrealistically large acceleration experienced by the space travelers during launch? Compare your answer with the free-fall acceleration 9.80 m/s^2.

20. A truck covers 40.0 m in 8.50 s while smoothly slowing down to a final speed of 2.80 m/s. (a) Find its original speed. (b) Find its acceleration.

$\boxed{21.}$ An object moving with uniform acceleration has a velocity of 12.0 cm/s in the positive x direction when its x coordinate is 3.00 cm. If its x coordinate 2.00 s later is -5.00 cm, what is its acceleration?

22. A 745i BMW car can brake to a stop in a distance of 121 ft. from a speed of 60.0 mi/h. To brake to a stop from a speed of 80.0 mi/h requires a stopping distance of 211 ft. What is the average braking acceleration for (a) 60 mi/h to rest, (b) 80 mi/h to rest, (c) 80 mi/h to 60 mi/h? Express the answers in mi/h/s and in m/s^2.

23. A speedboat moving at 30.0 m/s approaches a no-wake buoy marker 100 m ahead. The pilot slows the boat with a constant acceleration of -3.50 m/s^2 by reducing the throttle. (a) How long does it take the boat to reach the buoy? (b) What is the velocity of the boat when it reaches the buoy?

24. Figure P2.24 represents part of the performance data of a car owned by a proud physics student. (a) Calculate from the graph the total distance traveled. (b) What distance does the car travel between the times $t = 10$ s and $t = 40$ s? (c) Draw a graph of its acceleration versus time between $t = 0$ and $t = 50$ s. (d) Write an equation for x as a function of time for each phase of the motion, represented by (i) $0a$, (ii) ab, (iii) bc. (e) What is the average velocity of the car between $t = 0$ and $t = 50$ s?

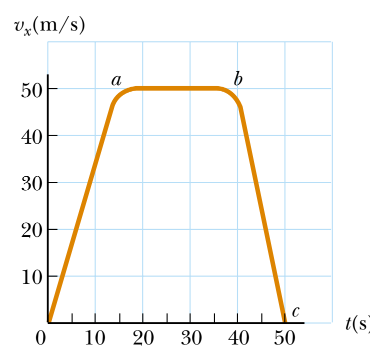


Figure P2.24

25. A particle moves along the x axis. Its position is given by the equation $x = 2 + 3t - 4t^2$ with x in meters and t in seconds. Determine (a) its position when it changes direction and (b) its velocity when it returns to the position it had at $t = 0$.

26. In the Daytona 500 auto race, a Ford Thunderbird and a Mercedes Benz are moving side by side down a straightaway at 71.5 m/s. The driver of the Thunderbird realizes he must make a pit stop, and he smoothly slows to a stop over a distance of 250 m. He spends 5.00 s in the pit and then accelerates out, reaching his previous speed of 71.5 m/s after a distance of 350 m. At this point, how far has the Thunderbird fallen behind the Mercedes Benz, which has continued at a constant speed?

$\boxed{\textbf{27.}}$ A jet plane lands with a speed of 100 m/s and can accelerate at a maximum rate of $-5.00\ \text{m/s}^2$ as it comes to rest. (a) From the instant the plane touches the runway, what is the minimum time interval needed before it can come to rest? (b) Can this plane land on a small tropical island airport where the runway is 0.800 km long?

28. A car is approaching a hill at 30.0 m/s when its engine suddenly fails just at the bottom of the hill. The car moves with a constant acceleration of $-2.00\ \text{m/s}^2$ while coasting up the hill. (a) Write equations for the position along the slope and for the velocity as functions of time, taking $x = 0$ at the bottom of the hill, where $v_i = 30.0$ m/s. (b) Determine the maximum distance the car rolls up the hill.

29. The driver of a car slams on the brakes when he sees a tree blocking the road. The car slows uniformly with an acceleration of $-5.60\ \text{m/s}^2$ for 4.20 s, making straight skid marks 62.4 m long ending at the tree. With what speed does the car then strike the tree?

30. *Help! One of our equations is missing!* We describe constant-acceleration motion with the variables and parameters v_{xi}, v_{xf}, a_x, t, and $x_f - x_i$. Of the equations in Table 2.2, the first does not involve $x_f - x_i$. The second does not contain a_x; the third omits v_{xf} and the last leaves out t. So to complete the set there should be an equation *not* involving v_{xi}. Derive it from the others. Use it to solve Problem 29 in one step.

$\boxed{\textbf{31.}}$ For many years Colonel John P. Stapp, USAF, held the world's land speed record. On March 19, 1954, he rode a rocket-propelled sled that moved down a track at a speed of 632 mi/h. He and the sled were safely brought to rest in 1.40 s (Fig. P2.31). Determine (a) the negative acceleration he experienced and (b) the distance he traveled during this negative acceleration.

32. A truck on a straight road starts from rest, accelerating at $2.00\ \text{m/s}^2$ until it reaches a speed of 20.0 m/s. Then the truck travels for 20.0 s at constant speed until the brakes are applied, stopping the truck in a uniform manner in an additional 5.00 s. (a) How long is the truck in motion? (b) What is the average velocity of the truck for the motion described?

33. An electron in a cathode ray tube (CRT) accelerates from 2.00×10^4 m/s to 6.00×10^6 m/s over 1.50 cm. (a) How long does the electron take to travel this 1.50 cm? (b) What is its acceleration?

34. In a 100-m linear accelerator, an electron is accelerated to 1.00% of the speed of light in 40.0 m before it coasts for 60.0 m to a target. (a) What is the electron's acceleration during the first 40.0 m? (b) How long does the total flight take?

35. Within a complex machine such as a robotic assembly line, suppose that one particular part glides along a straight track. A control system measures the average velocity of the part during each successive interval of time $\Delta t_0 = t_0 - 0$, compares it with the value v_c it should be, and switches a servo motor on and off to give the part a correcting pulse of acceleration. The pulse consists of a constant acceleration a_m applied for time interval $\Delta t_m = t_m - 0$ within the next control time interval Δt_0. As shown in Fig. P2.35, the part may be modeled as having zero acceleration when the motor is off (between t_m and t_0). A computer in the control system chooses the size of the acceleration so that the final velocity of the part will have the correct value v_c. Assume the part is initially at rest and is to have instantaneous velocity v_c at time t_0. (a) Find the required value of a_m in terms of v_c and t_m. (b) Show that the displacement Δx of the part during the time interval Δt_0 is given by $\Delta x = v_c\,(t_0 - 0.5t_m)$. For specified values of v_c and t_0, (c) what is the minimum displacement of the part? (d) What is the maximum displacement of the part? (e) Are both the minimum and maximum displacements physically attainable?

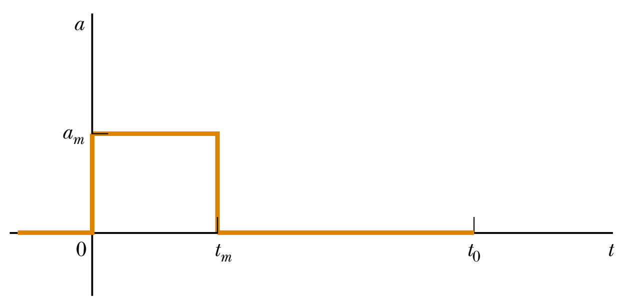


Figure P2.35

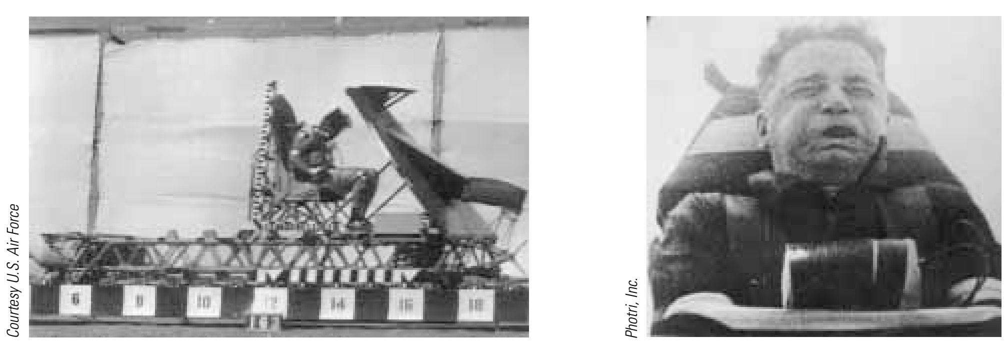


Figure P2.31 *(Left)* Col. John Stapp on rocket sled. *(Right)* Col. Stapp's face is contorted by the stress of rapid negative acceleration.

36. A glider on an air track carries a flag of length ℓ through a stationary photogate, which measures the time interval Δt_d during which the flag blocks a beam of infrared light passing across the photogate. The ratio $v_d = \ell/\Delta t_d$ is the average velocity of the glider over this part of its motion. Suppose the glider moves with constant acceleration. (a) Argue for or against the idea that v_d is equal to the instantaneous velocity of the glider when it is halfway through the photogate in space. (b) Argue for or against the idea that v_d is equal to the instantaneous velocity of the glider when it is halfway through the photogate in time.

37. A ball starts from rest and accelerates at 0.500 m/s^2 while moving down an inclined plane 9.00 m long. When it reaches the bottom, the ball rolls up another plane, where, after moving 15.0 m, it comes to rest. (a) What is the speed of the ball at the bottom of the first plane? (b) How long does it take to roll down the first plane? (c) What is the acceleration along the second plane? (d) What is the ball's speed 8.00 m along the second plane?

38. Speedy Sue, driving at 30.0 m/s, enters a one-lane tunnel. She then observes a slow-moving van 155 m ahead traveling at 5.00 m/s. Sue applies her brakes but can accelerate only at -2.00 m/s^2 because the road is wet. Will there be a collision? If yes, determine how far into the tunnel and at what time the collision occurs. If no, determine the distance of closest approach between Sue's car and the van.

39. Solve Example 2.8, "Watch out for the Speed Limit!" by a graphical method. On the same graph plot position versus time for the car and the police officer. From the intersection of the two curves read the time at which the trooper overtakes the car.

Section 2.6 Freely Falling Objects

> *Note:* In all problems in this section, ignore the effects of air resistance.

40. A golf ball is released from rest from the top of a very tall building. Neglecting air resistance, calculate (a) the position and (b) the velocity of the ball after 1.00, 2.00, and 3.00 s.

41. *Every morning at seven o'clock*
There's twenty terriers drilling on the rock.
The boss comes around and he says, "Keep still
And bear down heavy on the cast-iron drill

And drill, ye terriers, drill." And drill, ye terriers, drill.
It's work all day for sugar in your tea
Down beyond the railway. And drill, ye terriers, drill.

The foreman's name was John McAnn.
By God, he was a blamed mean man.
One day a premature blast went off
And a mile in the air went big Jim Goff. And drill ...

Then when next payday came around
Jim Goff a dollar short was found.
When he asked what for, came this reply:
"You were docked for the time you were up in the sky." And drill...
—American folksong

What was Goff's hourly wage? State the assumptions you make in computing it.

42. A ball is thrown directly downward, with an initial speed of 8.00 m/s, from a height of 30.0 m. After what time interval does the ball strike the ground?

43. A student throws a set of keys vertically upward to her sorority sister, who is in a window 4.00 m above. The keys are caught 1.50 s later by the sister's outstretched hand. (a) With what initial velocity were the keys thrown? (b) What was the velocity of the keys just before they were caught?

44. Emily challenges her friend David to catch a dollar bill as follows. She holds the bill vertically, as in Figure P2.44, with the center of the bill between David's index finger and thumb. David must catch the bill after Emily releases it without moving his hand downward. If his reaction time is 0.2 s, will he succeed? Explain your reasoning.

George Semple

Figure P2.44

45. In Mostar, Bosnia, the ultimate test of a young man's courage once was to jump off a 400-year-old bridge (now destroyed) into the River Neretva, 23.0 m below the bridge. (a) How long did the jump last? (b) How fast was the diver traveling upon impact with the water? (c) If the speed of sound in air is 340 m/s, how long after the diver took off did a spectator on the bridge hear the splash?

46. A ball is dropped from rest from a height h above the ground. Another ball is thrown vertically upwards from the ground at the instant the first ball is released. Determine the speed of the second ball if the two balls are to meet at a height $h/2$ above the ground.

47. A baseball is hit so that it travels straight upward after being struck by the bat. A fan observes that it takes 3.00 s for the ball to reach its maximum height. Find (a) its initial velocity and (b) the height it reaches.

48. It is possible to shoot an arrow at a speed as high as 100 m/s. (a) If friction is neglected, how high would an arrow launched at this speed rise if shot straight up? (b) How long would the arrow be in the air?

49. A daring ranch hand sitting on a tree limb wishes to drop vertically onto a horse galloping under the tree. The constant speed of the horse is 10.0 m/s, and the distance from the limb to the level of the saddle is 3.00 m. (a) What must be the horizontal distance between the saddle and limb when the ranch hand makes his move? (b) How long is he in the air?

50. A woman is reported to have fallen 144 ft from the 17th floor of a building, landing on a metal ventilator box, which she crushed to a depth of 18.0 in. She suffered only minor injuries. Neglecting air resistance, calculate (a) the speed of the woman just before she collided with the ventilator, (b) her average acceleration while in contact with the box, and (c) the time it took to crush the box.

51. The height of a helicopter above the ground is given by $h = 3.00t^3$, where h is in meters and t is in seconds. After 2.00 s, the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground?

52. A freely falling object requires 1.50 s to travel the last 30.0 m before it hits the ground. From what height above the ground did it fall?

Section 2.7 Kinematic Equations Derived from Calculus

53. Automotive engineers refer to the time rate of change of acceleration as the "jerk." If an object moves in one dimension such that its jerk J is constant, (a) determine expressions for its acceleration $a_x(t)$, velocity $v_x(t)$, and position $x(t)$, given that its initial acceleration, velocity, and position are a_{xi}, v_{xi}, and x_i, respectively. (b) Show that $a_x^2 = a_{xi}^2 + 2J(v_x - v_{xi})$.

54. A student drives a moped along a straight road as described by the velocity-versus-time graph in Figure P2.54. Sketch this graph in the middle of a sheet of graph paper. (a) Directly above your graph, sketch a graph of the position versus time, aligning the time coordinates of the two graphs. (b) Sketch a graph of the acceleration versus time directly below the v_x-t graph, again aligning the time coordinates. On each graph, show the numerical values of x and a_x for all points of inflection. (c) What is the acceleration at $t = 6$ s? (d) Find the position (relative to the starting point) at $t = 6$ s. (e) What is the moped's final position at $t = 9$ s?

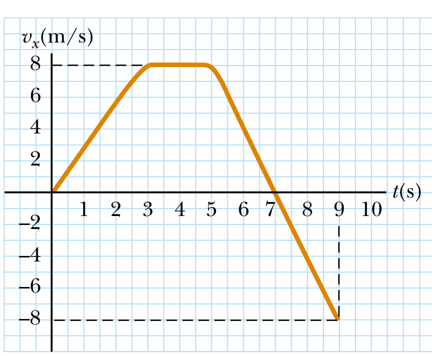


Figure P2.54

55. The speed of a bullet as it travels down the barrel of a rifle toward the opening is given by $v = (-5.00 \times 10^7)t^2 + (3.00 \times 10^5)t$, where v is in meters per second and t is in seconds. The acceleration of the bullet just as it leaves the barrel is zero. (a) Determine the acceleration and position of the bullet as a function of time when the bullet is in the barrel. (b) Determine the length of time the bullet is accelerated. (c) Find the speed at which the bullet leaves the barrel. (d) What is the length of the barrel?

56. The acceleration of a marble in a certain fluid is proportional to the speed of the marble squared, and is given (in SI units) by $a = -3.00v^2$ for $v > 0$. If the marble enters this fluid with a speed of 1.50 m/s, how long will it take before the marble's speed is reduced to half of its initial value?

Additional Problems

57. A car has an initial velocity v_0 when the driver sees an obstacle in the road in front of him. His reaction time is Δt_r, and the braking acceleration of the car is a. Show that the total stopping distance is

$$s_{\text{stop}} = v_0 \Delta t_r - v_0^2/2a.$$

Remember that a is a negative number.

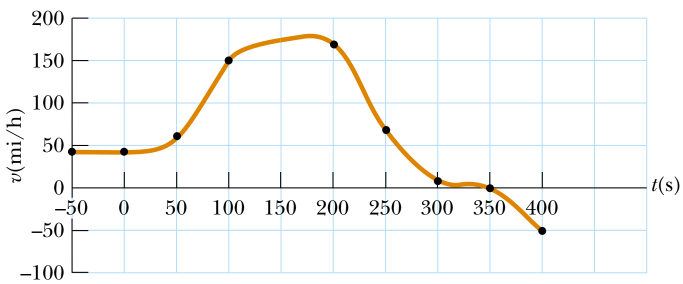
58. The yellow caution light on a traffic signal should stay on long enough to allow a driver to either pass through the intersection or safely stop before reaching the intersection. A car can stop if its distance from the intersection is greater than the stopping distance found in the previous problem. If the car is less than this stopping distance from the intersection, the yellow light should stay on long enough to allow the car to pass entirely through the intersection. (a) Show that the yellow light should stay on for a time interval

$$\Delta t_{\text{light}} = \Delta t_r - (v_0/2a) + (s_i/v_0)$$

where Δt_r is the driver's reaction time, v_0 is the velocity of the car approaching the light at the speed limit, a is the braking acceleration, and s_i is the width of the intersection. (b) As city traffic planner, you expect cars to approach an intersection 16.0 m wide with a speed of 60.0 km/h. Be cautious and assume a reaction time of 1.10 s to allow for a driver's indecision. Find the length of time the yellow light should remain on. Use a braking acceleration of -2.00 m/s^2.

59. The Acela is the Porsche of American trains. Shown in Figure P2.59a, the electric train whose name is pronounced ah-SELL-ah is in service on the Washington-New York-Boston run. With two power cars and six coaches, it can carry 304 passengers at 170 mi/h. The carriages tilt as much as 6° from the vertical to prevent passengers from feeling pushed to the side as they go around curves. Its braking mechanism uses electric generators to recover its energy of motion. A velocity-time graph for the Acela is shown in Figure P2.59b. (a) Describe the motion of the train in each successive time interval. (b) Find the peak positive acceleration of the train in the motion graphed. (c) Find the train's displacement in miles between $t = 0$ and $t = 200$ s.

(a)



(b)

Figure P2.59 (a) The Acela—1 171 000 lb of cold steel thundering along at 150 mi/h. (b) Velocity-versus-time graph for the Acela.

60. Liz rushes down onto a subway platform to find her train already departing. She stops and watches the cars go by. Each car is 8.60 m long. The first moves past her in 1.50 s and the second in 1.10 s. Find the constant acceleration of the train.

61. A dog's hair has been cut and is now getting 1.04 mm longer each day. With winter coming on, this rate of hair growth is steadily increasing, by 0.132 mm/day every week. By how much will the dog's hair grow during 5 weeks?

62. A test rocket is fired vertically upward from a well. A catapult gives it an initial speed of 80.0 m/s at ground level. Its engines then fire and it accelerates upward at 4.00 m/s^2 until it reaches an altitude of 1 000 m. At that point its engines fail and the rocket goes into free fall, with an acceleration of -9.80 m/s^2. (a) How long is the rocket in motion above the ground? (b) What is its maximum altitude? (c) What is its velocity just before it collides with the Earth? (You will need to consider the motion while the engine is operating separate from the free-fall motion.)

63. A motorist drives along a straight road at a constant speed of 15.0 m/s. Just as she passes a parked motorcycle police officer, the officer starts to accelerate at 2.00 m/s^2 to overtake her. Assuming the officer maintains this acceleration, (a) determine the time it takes the police officer to reach the motorist. Find (b) the speed and (c) the total displacement of the officer as he overtakes the motorist.

64. In Figure 2.10b, the area under the velocity versus time curve and between the vertical axis and time t (vertical dashed line) represents the displacement. As shown, this area consists of a rectangle and a triangle. Compute their areas and compare the sum of the two areas with the expression on the right-hand side of Equation 2.12.

65. Setting a new world record in a 100-m race, Maggie and Judy cross the finish line in a dead heat, both taking 10.2 s. Accelerating uniformly, Maggie took 2.00 s and Judy 3.00 s to attain maximum speed, which they maintained for the rest of the race. (a) What was the acceleration of each sprinter? (b) What were their respective maximum speeds? (c) Which sprinter was ahead at the 6.00-s mark, and by how much?

66. A commuter train travels between two downtown stations. Because the stations are only 1.00 km apart, the train never reaches its maximum possible cruising speed. During rush hour the engineer minimizes the time interval Δt between two stations by accelerating for a time interval Δt_1 at a rate $a_1 = 0.100$ m/s^2 and then immediately braking with acceleration $a_2 = -0.500$ m/s^2 for a time interval Δt_2. Find the minimum time interval of travel Δt and the time interval Δt_1.

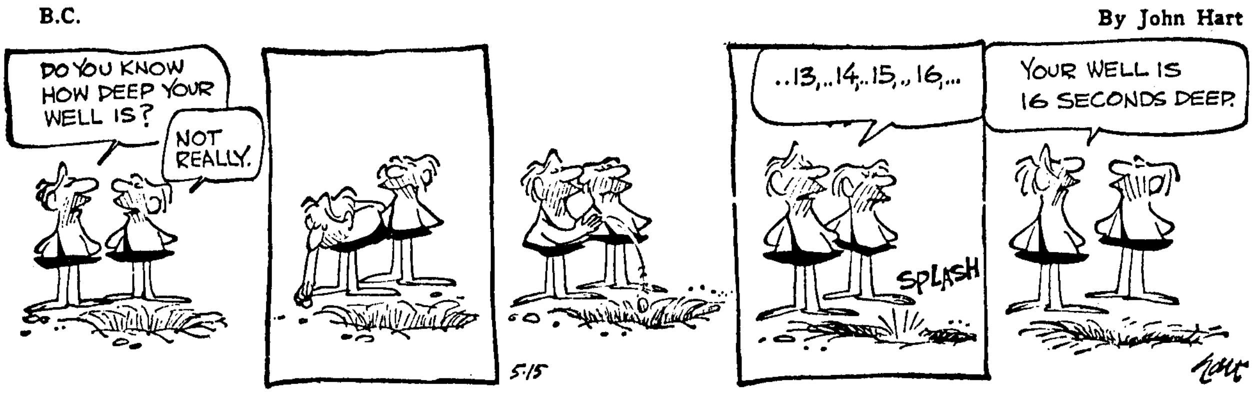
67. A hard rubber ball, released at chest height, falls to the pavement and bounces back to nearly the same height. When it is in contact with the pavement, the lower side of the ball is temporarily flattened. Suppose that the maximum depth of the dent is on the order of 1 cm. Compute an order-of-magnitude estimate for the maximum acceleration of the ball while it is in contact with the pavement. State your assumptions, the quantities you estimate, and the values you estimate for them.

68. At NASA's John H. Glenn research center in Cleveland, Ohio, free-fall research is performed by dropping experiment packages from the top of an evacuated shaft 145 m high. Free fall imitates the so-called microgravity environment of a satellite in orbit. (a) What is the maximum time interval for free fall if an experiment package were to fall the entire 145 m? (b) Actual NASA specifications allow for a 5.18-s drop time interval. How far do the packages drop and (c) what is their speed at 5.18 s? (d) What constant acceleration would be required to stop an experiment package in the distance remaining in the shaft after its 5.18-s fall?

69. An inquisitive physics student and mountain climber climbs a 50.0-m cliff that overhangs a calm pool of water. He throws two stones vertically downward, 1.00 s apart, and observes that they cause a single splash. The first stone has an initial speed of 2.00 m/s. (a) How long after release of the first stone do the two stones hit the water? (b) What initial velocity must the second stone have if they are to hit simultaneously? (c) What is the speed of each stone at the instant the two hit the water?

70. A rock is dropped from rest into a well. The well is not really 16 seconds deep, as in Figure P2.70. (a) The sound of the splash is actually heard 2.40 s after the rock is released from rest. How far below the top of the well is the surface of the water? The speed of sound in air (at the ambient temperature) is 336 m/s. (b) **What If?** If the travel time for the sound is neglected, what percentage error is introduced when the depth of the well is calculated?

71. To protect his food from hungry bears, a boy scout raises his food pack with a rope that is thrown over a tree limb at height h above his hands. He walks away from the vertical rope with constant velocity v_{boy}, holding the free end of the rope in his hands (Fig. P2.71). (a) Show that the speed v of the food pack is given by $x(x^2 + h^2)^{-1/2}\, v_{boy}$ where x



By permission of John Hart and Creators Syndicate, Inc.

Figure P2.70

is the distance he has walked away from the vertical rope. (b) Show that the acceleration a of the food pack is $h^2(x^2 + h^2)^{-3/2}\, v_{\text{boy}}^2$. (c) What values do the acceleration a and velocity v have shortly after he leaves the point under the pack ($x = 0$)? (d) What values do the pack's velocity and acceleration approach as the distance x continues to increase?

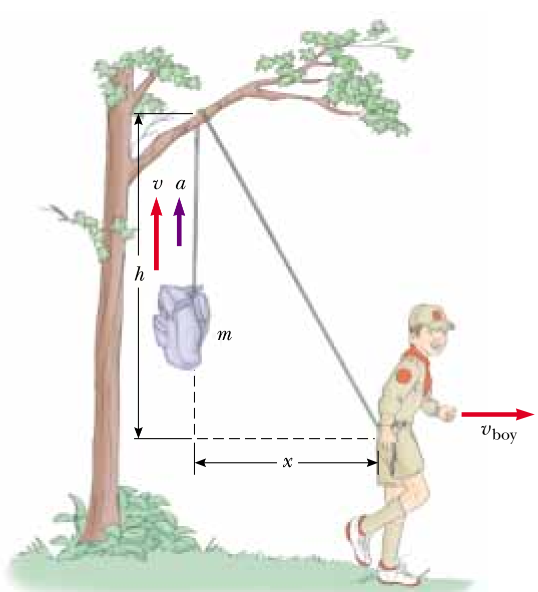


Figure P2.71 Problems 71 and 72.

72. In Problem 71, let the height h equal 6.00 m and the speed v_{boy} equal 2.00 m/s. Assume that the food pack starts from rest. (a) Tabulate and graph the speed–time graph. (b) Tabulate and graph the acceleration-time graph. Let the range of time be from 0 s to 5.00 s and the time intervals be 0.500 s.

73. Kathy Kool buys a sports car that can accelerate at the rate of 4.90 m/s^2. She decides to test the car by racing with another speedster, Stan Speedy. Both start from rest, but experienced Stan leaves the starting line 1.00 s before Kathy. If Stan moves with a constant acceleration of 3.50 m/s^2 and Kathy maintains an acceleration of 4.90 m/s^2, find (a) the time at which Kathy overtakes Stan, (b) the distance she travels before she catches him, and (c) the speeds of both cars at the instant she overtakes him.

74. Astronauts on a distant planet toss a rock into the air. With the aid of a camera that takes pictures at a steady rate, they record the height of the rock as a function of time as given in Table P2.74. (a) Find the average velocity of the rock in the time interval between each measurement and the next. (b) Using these average velocities to approximate instantaneous velocities at the midpoints of the time intervals, make a graph of velocity as a function of time. Does the rock move with constant acceleration? If so, plot a straight line of best fit on the graph and calculate its slope to find the acceleration.

Table P2.74

Height of a Rock versus Time

Time (s)	Height (m)	Time (s)	Height (m)
0.00	5.00	2.75	7.62
0.25	5.75	3.00	7.25
0.50	6.40	3.25	6.77
0.75	6.94	3.50	6.20
1.00	7.38	3.75	5.52
1.25	7.72	4.00	4.73
1.50	7.96	4.25	3.85
1.75	8.10	4.50	2.86
2.00	8.13	4.75	1.77
2.25	8.07	5.00	0.58
2.50	7.90		

75. Two objects, A and B, are connected by a rigid rod that has a length L. The objects slide along perpendicular guide rails, as shown in Figure P2.75. If A slides to the left with a constant speed v, find the velocity of B when $\alpha = 60.0°$.

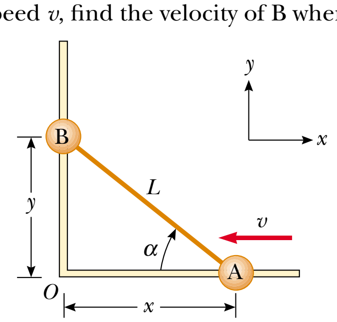


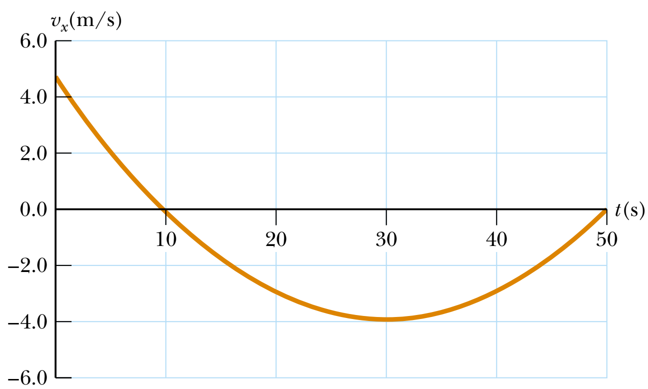
Figure P2.75

Answers to Quick Quizzes

2.1 (c). If the particle moves along a line without changing direction, the displacement and distance traveled over any time interval will be the same. As a result, the magnitude of the average velocity and the average speed will be the same. If the particle reverses direction, however, the displacement will be less than the distance traveled. In turn, the magnitude of the average velocity will be smaller than the average speed.

2.2 (b). If the car is slowing down, a force must be pulling in the direction opposite to its velocity.

2.3 False. Your graph should look something like the following. This v_x-t graph shows that the maximum speed is about 5.0 m/s, which is 18 km/h (= 11 mi/h), and so the driver was not speeding.



Answer to Quick Quiz 2.3

2.4 (c). If a particle with constant acceleration stops and its acceleration remains constant, it must begin to move again in the opposite direction. If it did not, the acceleration would change from its original constant value to zero. Choice (a) is not correct because the direction of acceleration is not specified by the direction of the velocity. Choice (b) is also not correct by counterexample—a car moving in the $-x$ direction and slowing down has a positive acceleration.

2.5 Graph (a) has a constant slope, indicating a constant acceleration; this is represented by graph (e).

Graph (b) represents a speed that is increasing constantly but not at a uniform rate. Thus, the acceleration must be increasing, and the graph that best indicates this is (d).

Graph (c) depicts a velocity that first increases at a constant rate, indicating constant acceleration. Then the velocity stops increasing and becomes constant, indicating zero acceleration. The best match to this situation is graph (f).

2.6 (e). For the entire time interval that the ball is in free fall, the acceleration is that due to gravity.

2.7 (d). While the ball is rising, it is slowing down. After reaching the highest point, the ball begins to fall and its speed increases.

2.8 (a). At the highest point, the ball is momentarily at rest, but still accelerating at $-g$.